CMPT 413/713: Natural Language Processing

Classification

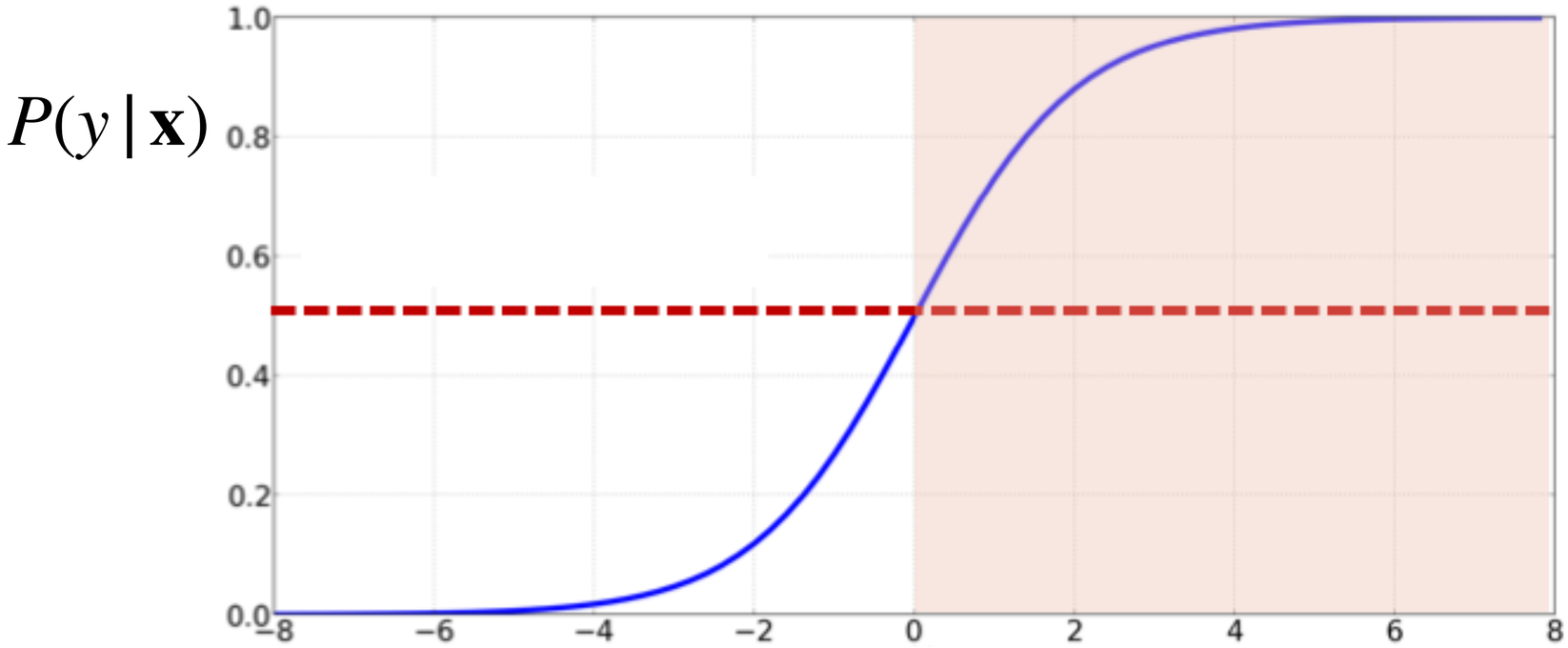
Spring 2024
2024-01-17

Adapted from slides from Danqi Chen, Karthik Narasimhan, and Anoop Sarkar

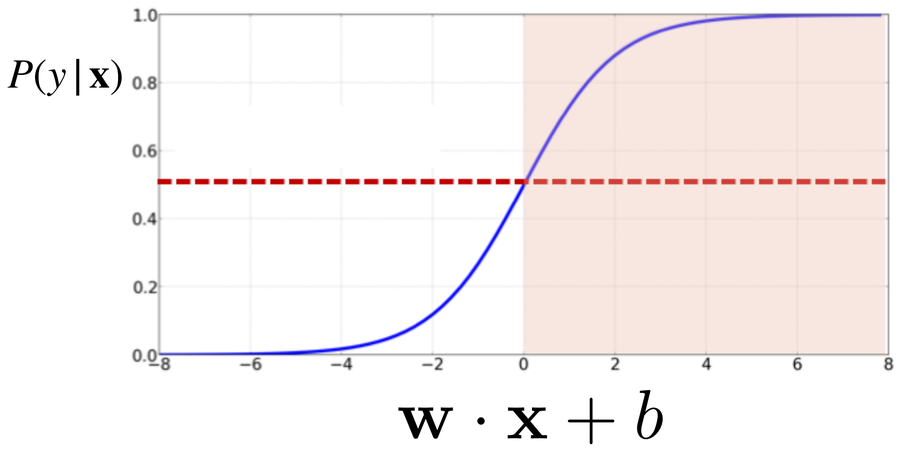
Logistic regression

Discriminative Model

- Logistic Regression: model $p(y \mid x)$ directly



Logistic Regression



- Powerful supervised model
- Baseline approach to most NLP tasks
- Connections with neural networks
- Binary (two classes) or multinomial (>2 classes)

Discriminative Model

discriminative model

- Logistic Regression: $\hat{c} = \arg\max_c P(c \mid d)$

focus on discriminating features

- Naive Bayes: $\hat{c} = \arg\max_c P(c)P(d \mid c)$

generative model

can be used to generate the cat and the dog

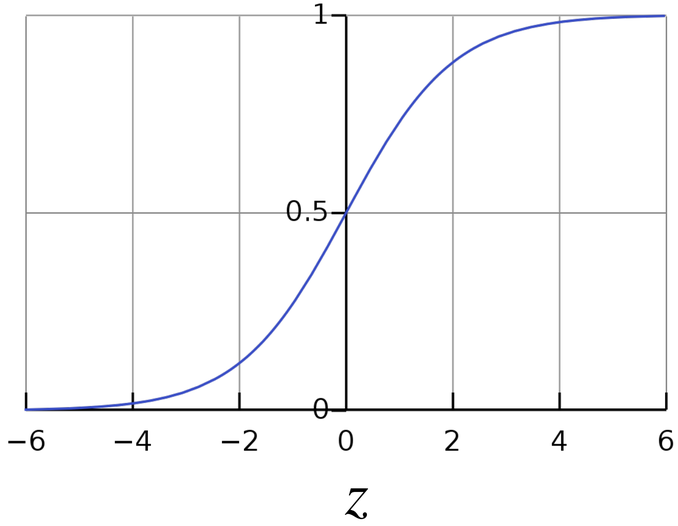
cats have whiskers

Classification function

- *Given*: Input feature vector $[x_1, x_2, \ldots, x_d]$
- *Output*: $P(y = 1 \mid x)$ and $P(y = 0 \mid x)$ *(binary classification)*
- Use a *function,* $F : \mathbb{R}^d \rightarrow [0,1]$ to model the probability $P(y \mid x)$
- Sigmoid:

$$P(y|x) = \frac{1}{1 + e^{-z}}$$

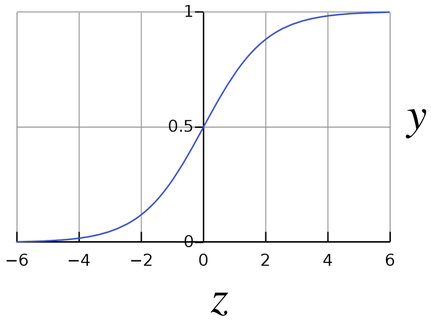
$$z = \sum_{i=1}^{d} w_i x_i + b$$



$P(y|x)$

Why the sigmoid?

$$y = \frac{1}{1 + e^{-z}}$$



- Binary logistic regression uses the log-odds to determine the probability.

- Log-odds:

$$\ell = \log \frac{p}{1-p}$$

Value is also known as logit (logistic unit)

$$\frac{p}{1-p} = \exp(\ell)$$

$$p = \exp(\ell)(1-p)$$

$$p(1 + \exp(\ell)) = \exp(\ell)$$

$$p = \frac{\exp(\ell)}{(1 + \exp(\ell))}$$

$$= \frac{1}{(1 + \exp(-\ell))} = \sigma(\ell)$$

- *z* is a linear estimate of the log-odds: $z = \sum_{i=1}^{d} w_i x_i + b$

Weights and bias

- *Which features are important* and *how much*?
- Learn a vector of **weights** and a **bias**
- Weights: Vector of real numbers, $\mathbf{w} = [w_1, w_2, \ldots, w_d]$
- Bias: Scalar intercept, b

$$z = \sum_{i=1}^{d} w_i x_i + b$$

Vector form

$$z = \mathbf{w} \cdot \mathbf{x} + b = \mathbf{w}^T \mathbf{x} + b$$

Putting it together

Given x, compute $z = \mathbf{w} \cdot \mathbf{x} + b$

Compute probabilities: $P(y = 1 \,|\, x) = \sigma(z) = \dfrac{1}{1 + e^{-z}}$

$$P(y = 1) = \sigma(\mathbf{w} \cdot \mathbf{x} + b)$$
$$\frac{1}{1 + e^{-(\mathbf{w} \cdot \mathbf{x} + b)}}$$

$$\begin{aligned} P(y = 0) &= 1 - \sigma(\mathbf{w} \cdot \mathbf{x} + b) \\ &= 1 - \frac{1}{1 + e^{-(\mathbf{w} \cdot \mathbf{x} + b)}} \\ &= \frac{e^{-(\mathbf{w} \cdot \mathbf{x} + b)}}{1 + e^{-(\mathbf{w} \cdot \mathbf{x} + b)}} \end{aligned}$$

Decision boundary:

$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1 \,|\, x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

Decision boundary

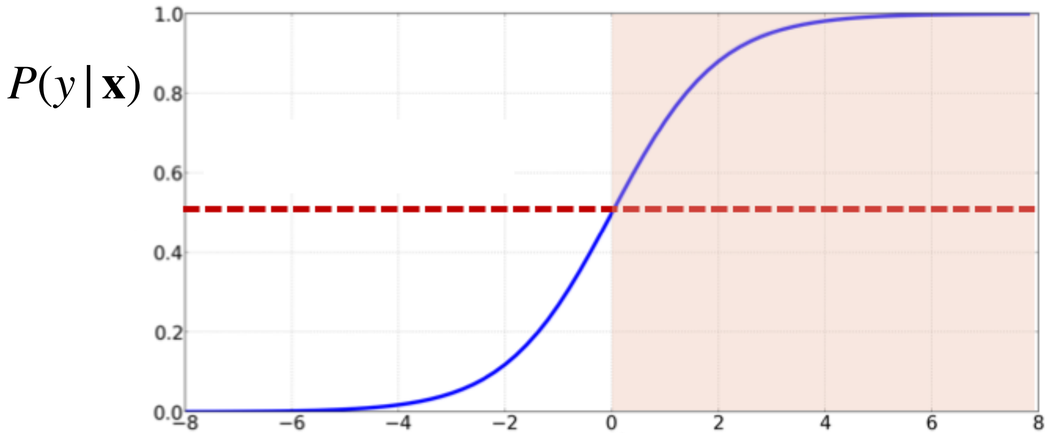
Threshold can be set as hyperparameter that is tuned

Decision boundary:

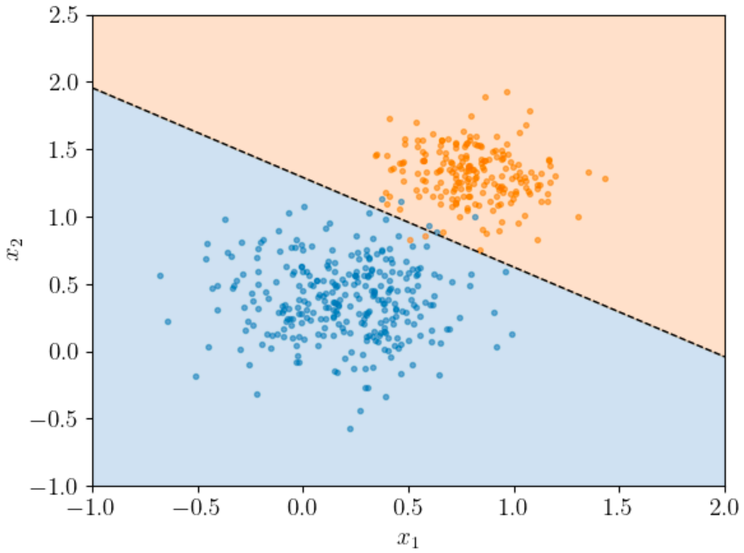
$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1 \mid x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{w} \cdot \mathbf{x} + b > 0$$

$$\mathbf{w} \cdot \mathbf{x} + b \leq 0$$



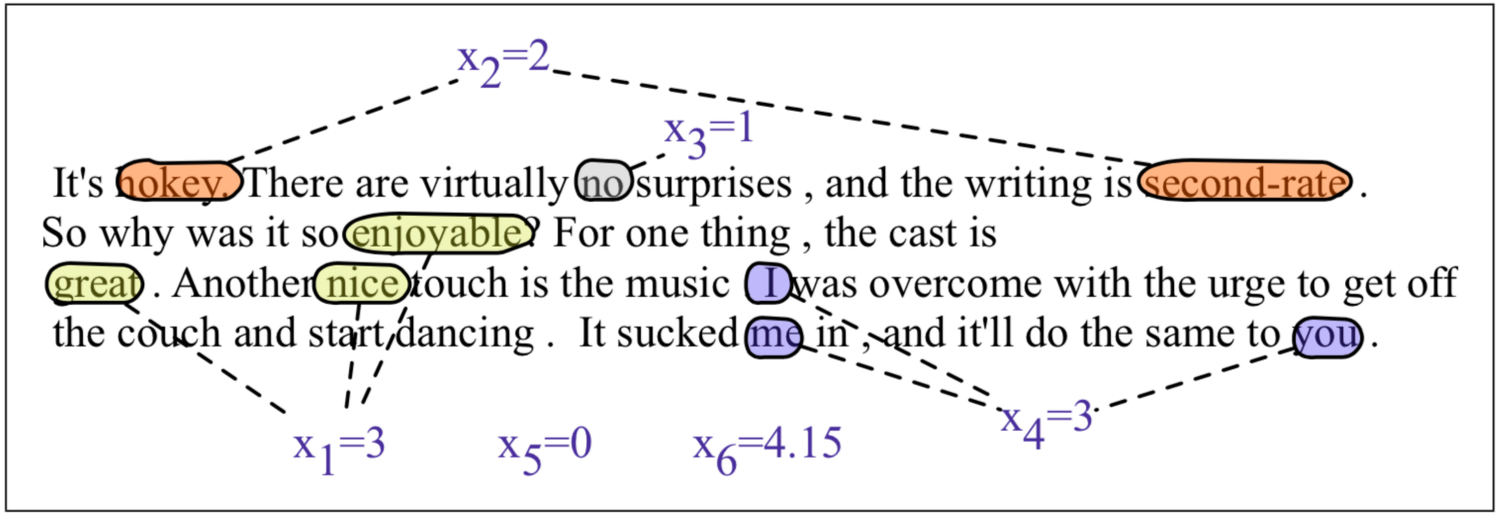
$$\mathbf{w} \cdot \mathbf{x} + b$$



Decision boundary is linear function of features

What do we use as features?

Example: Sentiment classification



Var	Definition	Value in Fig. 5.2
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if ``no'' } \in \text{ doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if ``!'' } \in \text{ doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(64) = 4.15$

(Credits: Dan Jurafsky)

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x_6	log(word count of doc)	$\ln(64) = 4.15$

- Assume weights $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$ and bias $b = 0.1$

$$
\begin{aligned}
p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\
&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.15] + 0.1) \\
&= \sigma(.805) \\
&= 0.69 \\
p(-|x) = P(Y=0|x) &= 1 - \sigma(w \cdot x + b) \\
&= 0.31
\end{aligned}
$$

Feature design

- Most important rule: Data is *key*!
- Linguistic intuition (e.g. part of speech tags, parse trees)
- Complex combinations

$$x_1 = \begin{cases} 1 & \text{if } \text{“}Case(w_i) = \text{Lower”} \\ 0 & \text{otherwise} \end{cases}$$

$$x_2 = \begin{cases} 1 & \text{if } \text{“}w_i \in \text{AcronymDict”} \\ 0 & \text{otherwise} \end{cases}$$

$$x_3 = \begin{cases} 1 & \text{if } \text{“}w_i = \text{St.} \ \& \ Case(w_{i-1}) = \text{Cap”} \\ 0 & \text{otherwise} \end{cases}$$

- Feature templates
  - Sparse representations, hash only seen features into index
  - Ex. Trigram(*logistic regression classifier*) = Feature #78
- Advanced: Representation learning (we will see this with neural networks!)

Multinomial Logistic Regression

Multinomial Logistic Regression

- What if we have more than 2 classes? (e.g. Part of speech tagging, named entity recognition)
- Need to model $P(y = c \mid x) \forall c \in C$
- Generalize **sigmoid** function to **softmax**

$$\text{softmax}(z_i) = \frac{e^{z_i}}{\sum_{j=1}^{k} e^{z_j}} \quad 1 \leq i \leq k$$

Normalization

Softmax

- Similar to sigmoid, softmax squashes values towards 0 or 1
- If $z = [0,1,2,3,4]$, then
  - softmax$(z) = ([0.0117,0.0317,0.0861,0.2341,0.6364])$
- For multinomial LR,

$$P(y = c \mid x) = \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_j}}$$

Features in multinomial LR

- Features need to include both input (x) and class (c)

- Implicit in binary case

Var	Definition	Wt
$f_1(0,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	-4.5
$f_1(+,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	2.6
$f_1(-,x)$	$\begin{cases} 1 & \text{if } \text{“!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1.3

Learning

- Generalize binary loss to multinomial CE loss:

$$L_{CE}(\hat{y}, y) = -\sum_{c=1}^{k} 1\{y = k\} \log P(y = k \mid x)$$

$$= -\sum_{c=1}^{k} 1\{y = k\} \log \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_c}}$$

Binary CE loss

$$-\sum_{i=1}^{n} [y \log \hat{y} + (1 - y) \log(1 - \hat{y})]$$

- Gradient:

$$\frac{dL_{CE}}{dw_c} = -(1\{y = c\} - P(y = c \mid x))x_c$$

$$= -\left(1\{y = c\} - \frac{e^{w_c \cdot x + b_c}}{\sum_{j=1}^{k} e^{w_j \cdot x + b_c}}\right) x_c$$

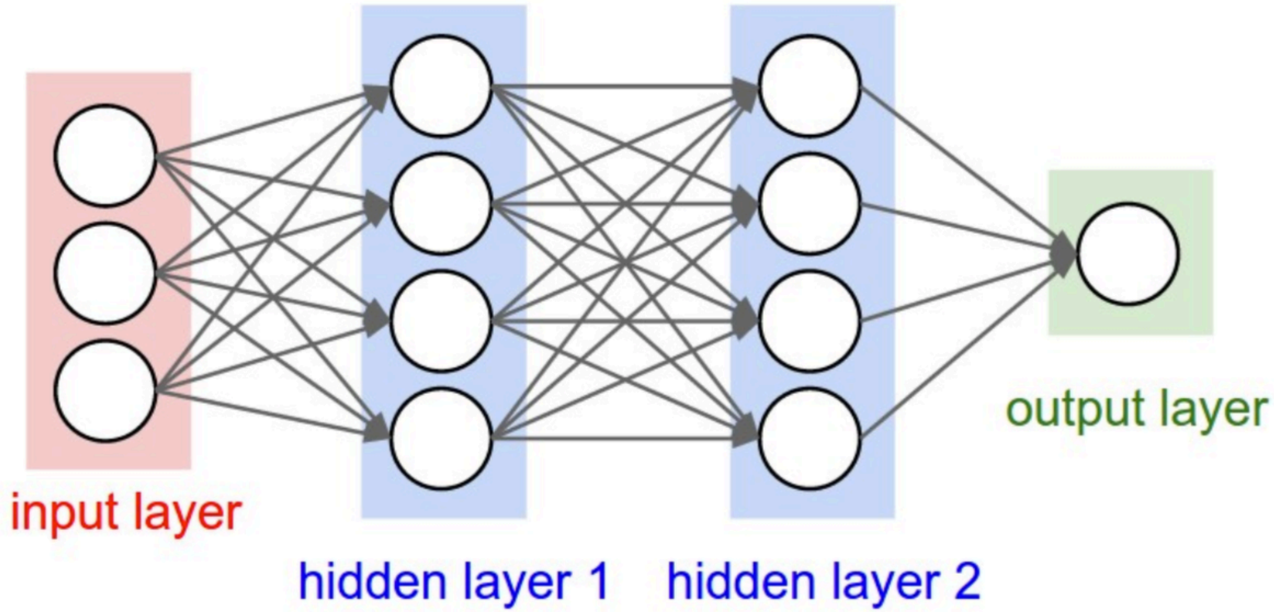
Logistic Regression: what's good and what's not

- More freedom in designing features
  - No strong independence assumptions like Naive Bayes
  - More robust to correlated features ("San Francisco" vs "Boston") —LR is likely to work better than NB
  - Can even have the same feature twice! (*why?*)
- **However**: not as good on small datasets (compared to Naive Bayes)
- Interpreting learned weights can be challenging

Relationship to neural networks

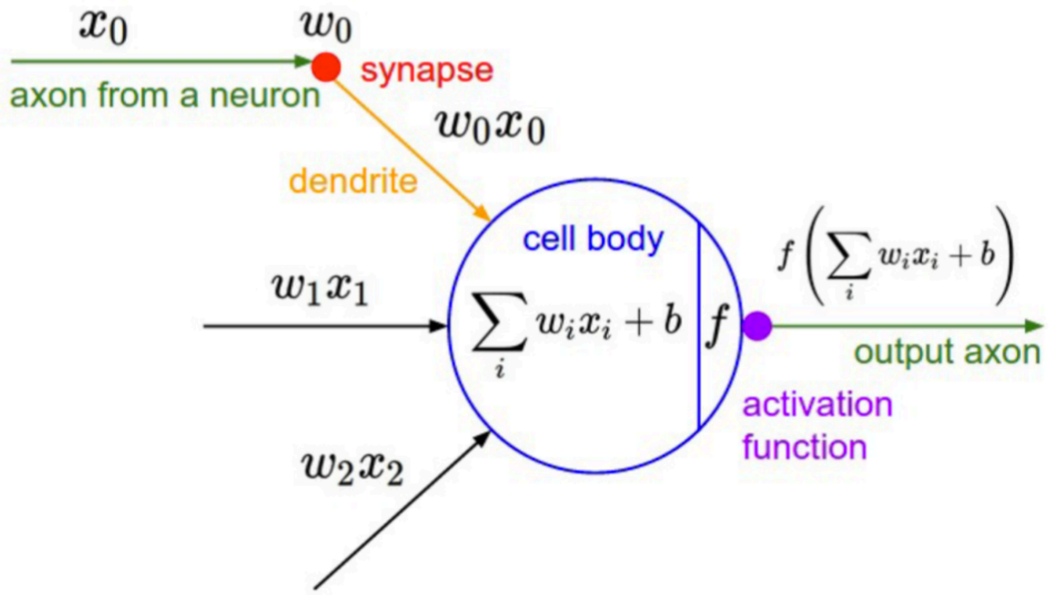
Feed-forward NNs

- Input: $x_1, \ldots, x_d$
- Output: $y \in \{0,1\}$



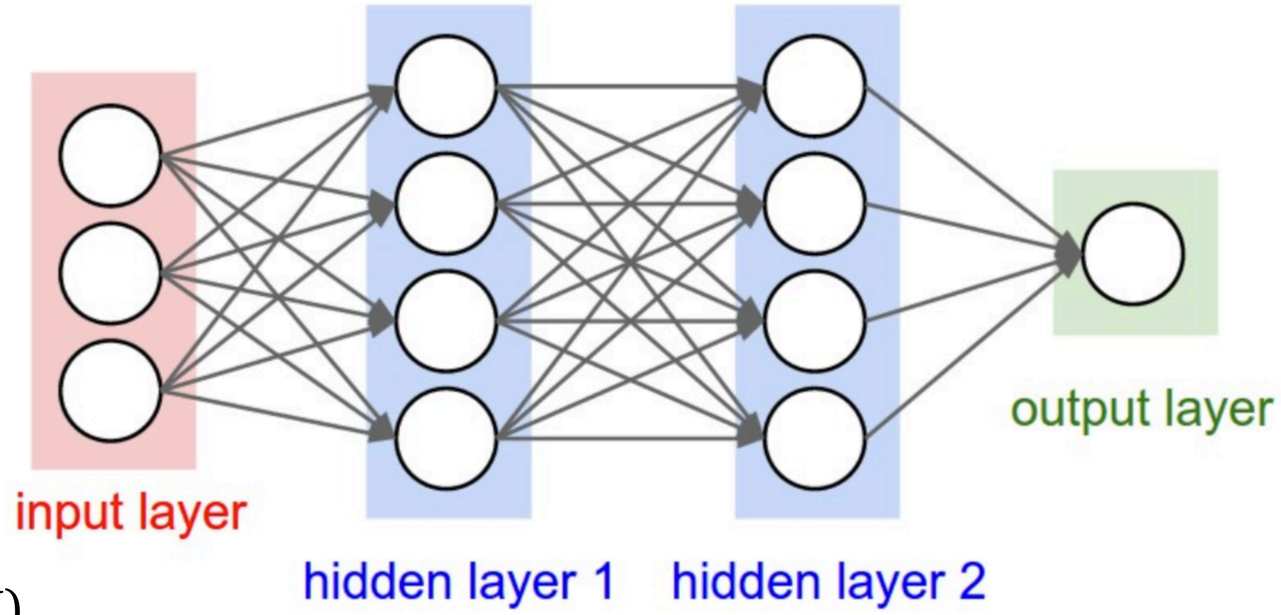
An artificial neuron

- A neuron is a computational unit that has scalar inputs and an output
- Each input has an associated weight.
- The neuron multiples each input by its weight, sums them, applied a **nonlinear function** to the result, and passes it to its output.



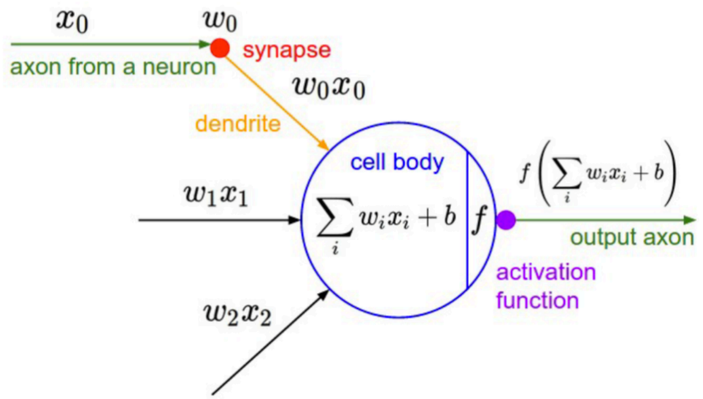
Neural networks

- The neurons are connected to each other, forming a **network**
- The output of a neuron may feed into the inputs of other neurons



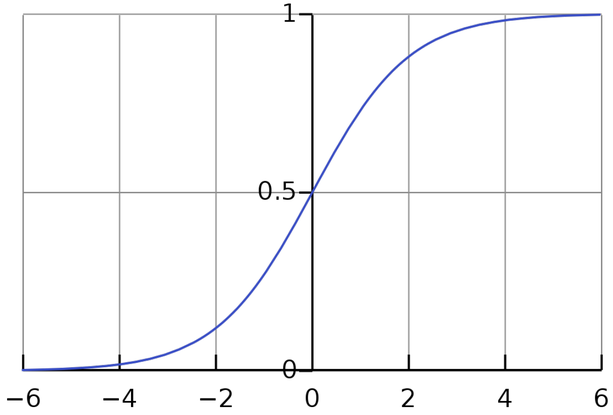
- Feed forward network (FFN)
- Fully connected network (FCN)

A neuron can be a binary logistic regression unit

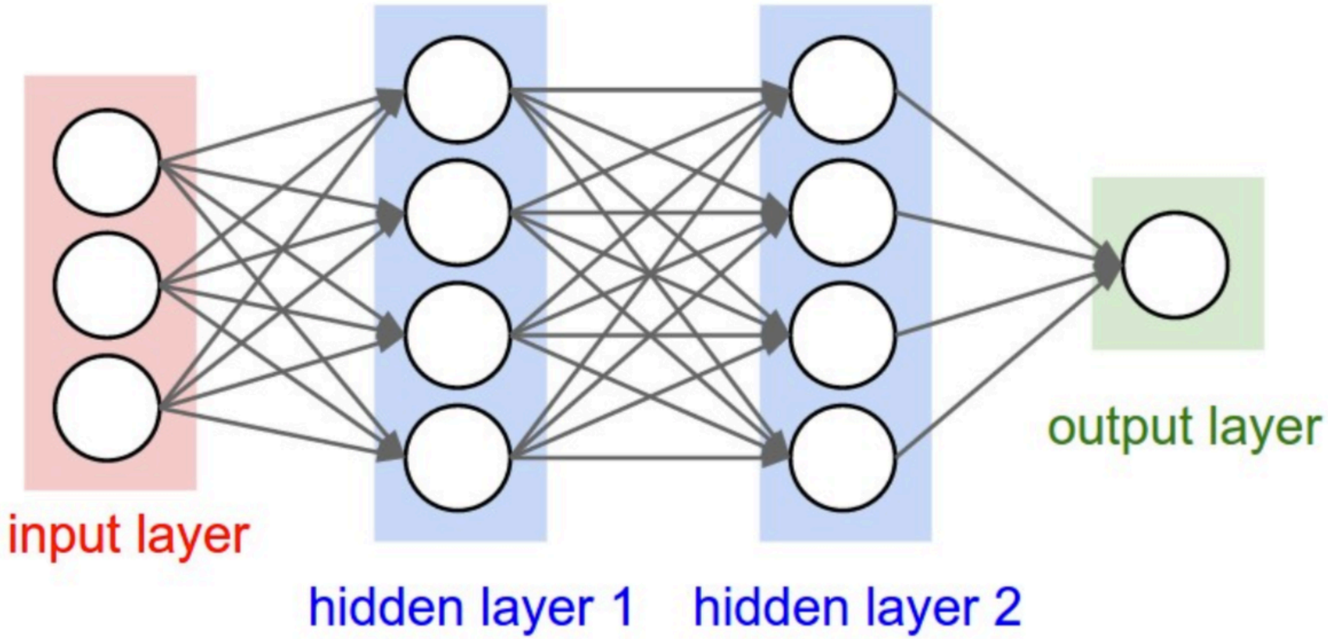


$$f(z) = \frac{1}{1 + e^{-z}}$$

$$h_{\mathbf{w},b}(\mathbf{x}) = f(\mathbf{w}^\intercal \mathbf{x} + b)$$



A neural network
= running several logistic regressions at the same time

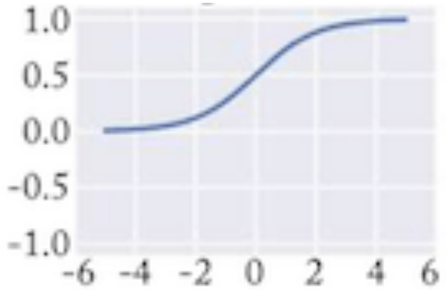


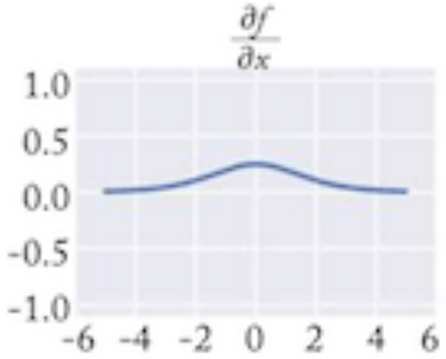
- If we feed a vector of inputs through a bunch of logistic regression functions, then we get a vector of outputs…
- which we can feed into another logistic regression function

NN has other activation functions

sigmoid

$$f(z) = \frac{1}{1 + e^{-z}}$$

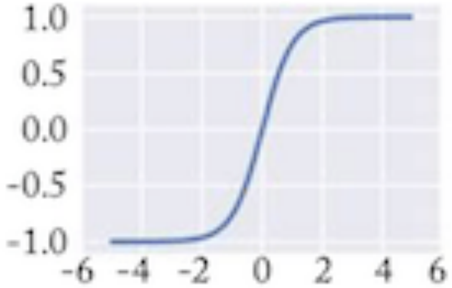


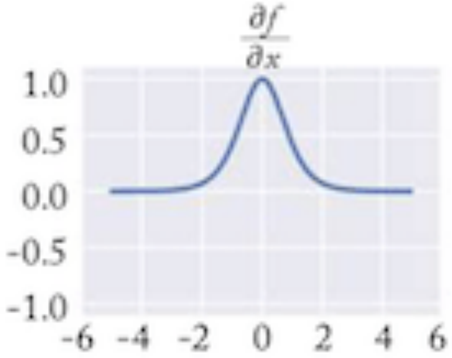


$$f'(z) = f(z) \times (1 - f(z))$$

Zero centered
tanh

$$f(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$



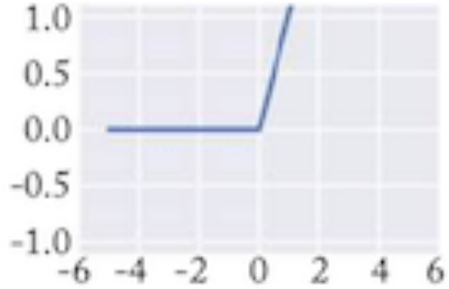


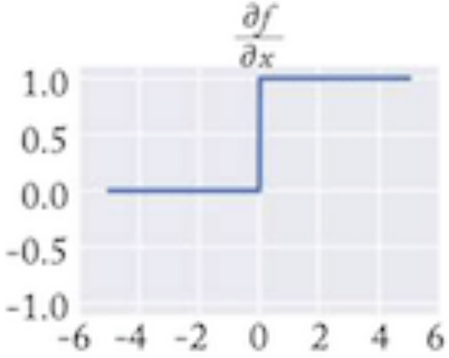
$$f'(z) = 1 - f(z)^2$$

Advantages of ReLU?

ReLU
(rectified linear unit)

$$f(z) = \max(0, z)$$





$$f'(z) = \begin{cases} 1 & z > 0 \\ 0 & z < 0 \end{cases}$$

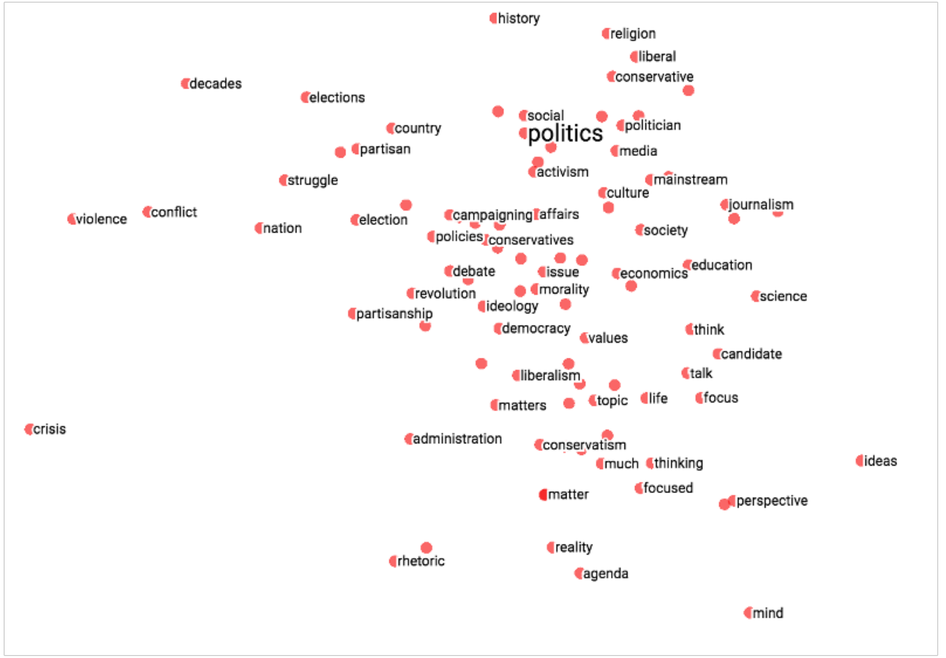
Advantages of neural networks

- Learn feature representations (no longer have to hand craft features)

Hand-crafted features

Var	Definition	Value in Fig. 5.2
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(64) = 4.15$

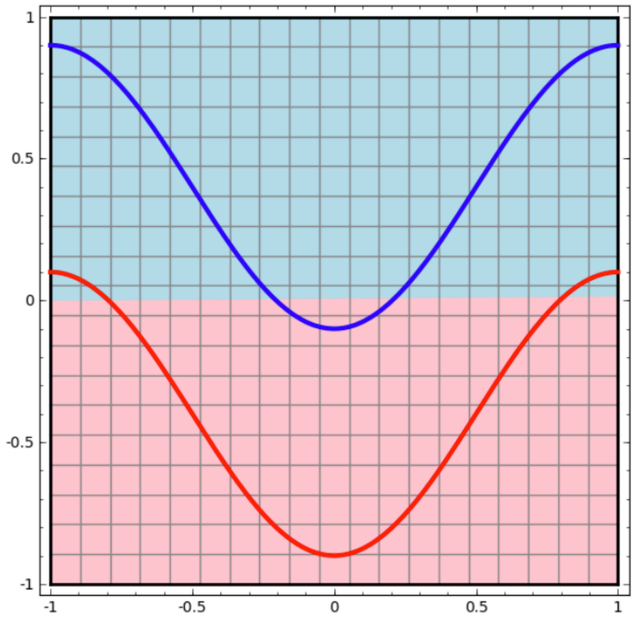
Learned representations



Advantages of neural networks

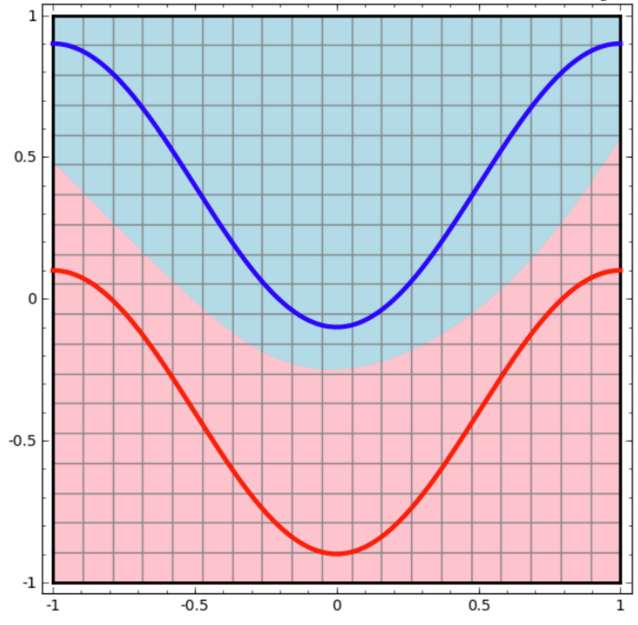
- Learn feature representations (no longer have to hand code features)
- Multiple layers allow for more complex functions with non-linear decision boundaries

Linear decision boundary



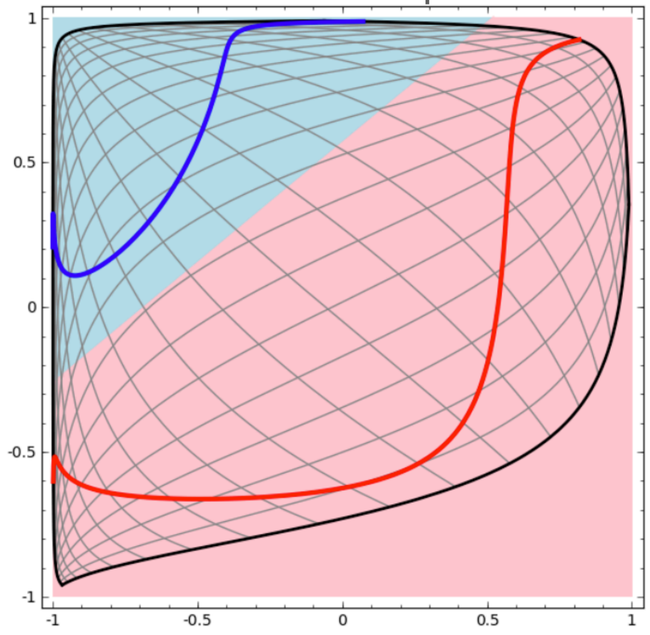
Linear classifier

Non-linear decision boundary



Neural Networks

Linear decision boundary
in transformed space



Hidden representations
are linearly separable!

Advantages of neural networks

- Learn feature representations (no longer have to hand code features)
- Multiple layers allow for more complex functions with non-linear decision boundaries
- Can build up complex models by connecting “neurons” into building blocks, and by connecting these building blocks together
- Optimization is mostly taken care of by auto-differentiation libraries
- Development of flexible deep learning libraries allow researchers and developers to focus more on modelling the problem

Back to Logistic Regression
How to learn the parameters?

Learning

- We have our **classification function -** how to assign weights and bias?

- Goal: predicted label $\hat{y}$ as close as possible to actual label y

  - **Distance metric/Loss function** between $\hat{y}$ and y : $L(\hat{y}, y)$

  - **Optimization algorithm** for updating weights

Loss function

- Assume $\hat{y} = \sigma(\mathbf{w} \cdot \mathbf{x} + b)$
- $L(\hat{y}, y) =$ Measure of difference between $\hat{y}$ and y. But what form?
- Maximum likelihood estimation (conditional):
  - Choose $\mathbf{w}$ and b such that $\log P(y \mid x)$ is maximized for true labels y paired with input x
  - Similar to language models!
    - $\max \log P(w_t \mid w_{t-n}, \ldots, w_{t-1})$ given a corpus

Cross Entropy loss

- Assume a single data point (x, y) and two classes
- Classifier probability: $P(y \mid x) = \hat{y}^{\,y}(1 - \hat{y})^{1-y}$
- Log probability: $\log P(y \mid x) = \log[\hat{y}^{\,y}(1 - \hat{y})^{1-y}]$

$$= y \log \hat{y} + (1 - y)\log(1 - \hat{y})$$

- Loss: $-\log P(y \mid x) = -\,[y \log \hat{y} + (1 - y)\log(1 - \hat{y})]$

$$y = 1 \Rightarrow -\log \hat{y} \qquad\qquad y = 0 \Rightarrow -\log(1 - \hat{y})$$

Example: Computing CE Loss

Var	Definition	Va
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(64) = 4.15$

- Assume weights $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$ and bias $b = 0.1$
- If y = 1 (positive sentiment), $L_{CE} = -\log(0.69) = 0.37$
- If y = 0 (negative sentiment), $L_{CE} = -\log(0.31) = 1.17$

Cross Entropy loss

- Assume *n* data points $(x^{(i)}, y^{(i)})$
- Classifier probability: $\prod_{i=1}^{n} P(y^{(i)} \mid x^{(i)}) = \prod_{i=1}^{n} \hat{y}^{y}(1-\hat{y})^{1-y}$
- Loss: $-\log \prod_{i=1}^{n} P(y^{(i)} \mid x^{(i)}) = -\sum_{i=1}^{n} \log P(y^{(i)} \mid x^{(i)})$

$$L_{CE} = -\sum_{i=1}^{n} [y \log \hat{y} + (1-y)\log(1-\hat{y})]$$

Maximizing likelihood = Minimizing cross entropy

Properties of CE Loss

- $L_{CE} = -\sum_{i=1}^{n} [y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$

- Ranges from 0 (perfect predictions) to ∞

- Lower the value, better the classifier

- *Cross-entropy* between the true distribution $P(y \mid x)$ and predicted distribution $P(\hat{y} \mid x)$

$$L_{CE} = -\frac{n_{y=1}}{n} \log \hat{y} - \frac{n_{y=0}}{n} \log(1 - \hat{y}) = \sum_{y=0}^{1} p(y \mid x) \log p(\hat{y} \mid x)$$

Optimization

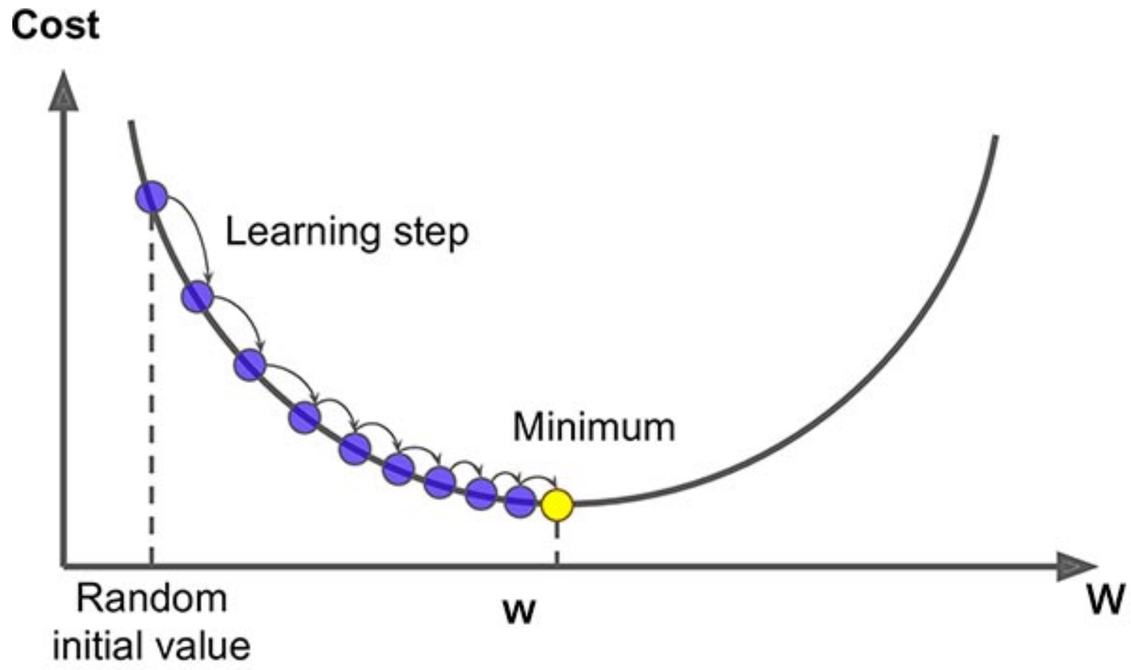
- We have our **classification function** and **loss function** - how do we find the best w and b?

$$\theta = [w; b]$$

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{n} \sum_{i=1}^{n} L_{CE}(y^{(i)}, x^{(i)}; \theta)$$

- **No close form solution**: need to numerically determine optimal solution
- Gradient descent:
  - Find direction of steepest slope
  - Move in the opposite direction

Gradient descent (1-D)

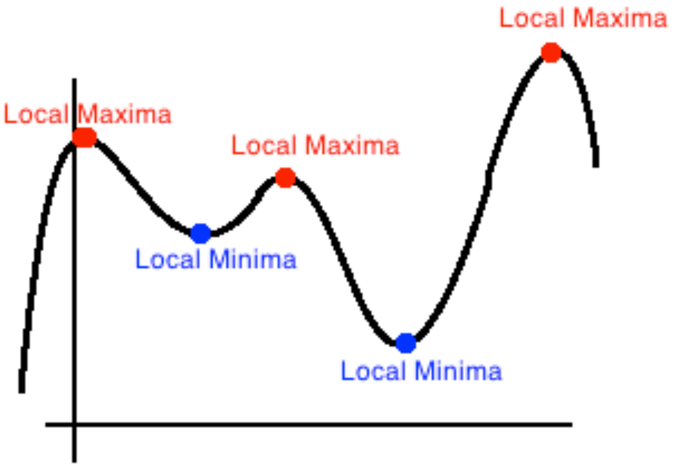


$$\theta^{t+1} = \theta^t - \eta \frac{d}{d\theta} L(f(x;\theta), y)$$

Loss function we are optimizing

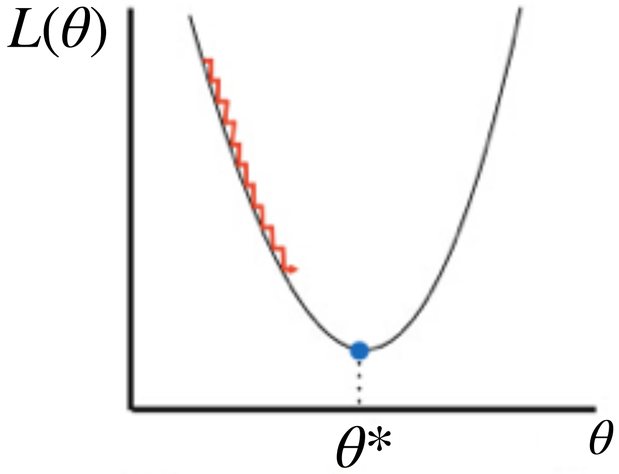
Gradient descent for LR

- Cross entropy loss for logistic regression is **convex** (i.e. has only one global minimum)
  - No local minima to get stuck in
- Deep neural networks are not so easy
  - Non-convex

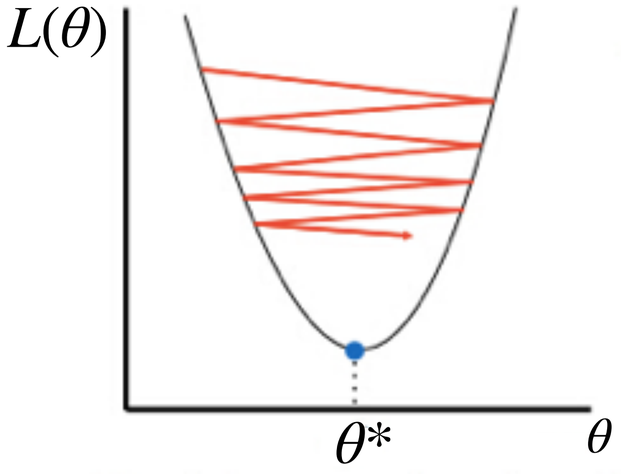


Learning Rate

- Updates: $\theta^{t+1} = \theta^t - \eta \frac{d}{d\theta} L(f(x;\theta), y)$
- Magnitude of movement along gradient
- Higher/faster learning rate = larger updates to parameters



Too small: converge very slowly



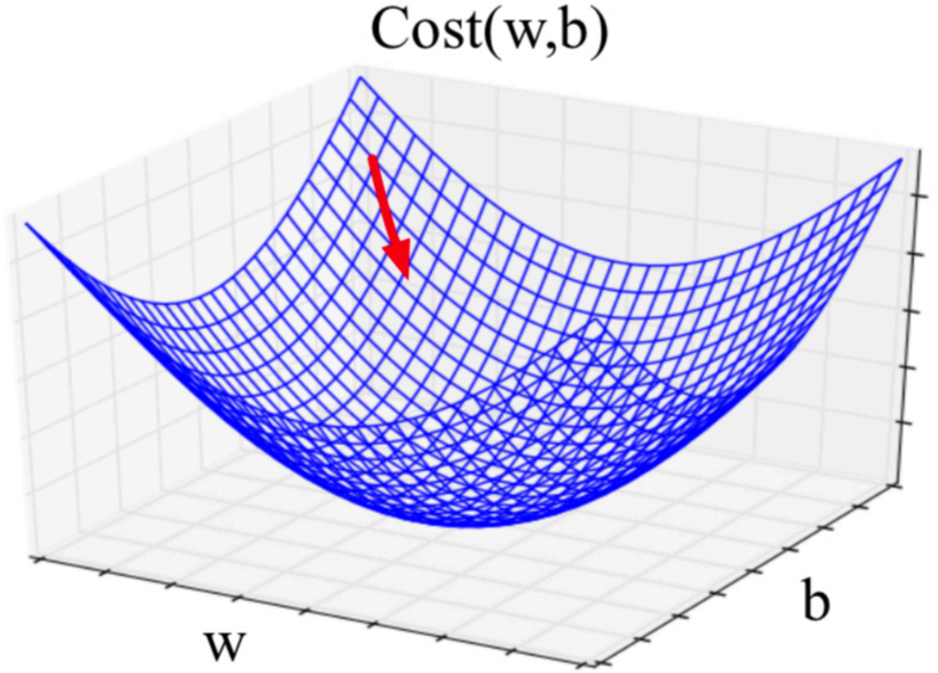
Too big: overshoot and even diverge

Gradient descent with vector weights

- In LR: weight w is a vector

- Express slope as a partial derivative of loss w.r.t each weight:

$$\nabla_\theta L(f(x;\theta),y)) = \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$



- Updates: $\theta^{(t+1)} = \theta^t - \eta \nabla L(f(x;\theta), y)$

Gradient for logistic regression

$$L_{CE} = -\sum_{i=1}^{n} [y^{(i)} \log \sigma(w \cdot x^{(i)} + b) + (1 - y^{(i)}) \log(1 - \sigma(w \cdot x^{(i)} + b))]$$

Gradient, $$\frac{dL_{CE}(w,b)}{dw_j} = \sum_{i=1}^{n} [\sigma(w \cdot x^{(i)} + b) - y^{(i)}] x_j^{(i)}$$

Diff between true y and prediction

input feature value

$$\frac{dL_{CE}(w,b)}{db} = \sum_{i=1}^{n} [\sigma(w \cdot x^{(i)} + b) - y^{(i)}]$$

Properties of the logistic function

- Relationship to $\sigma(-z)$

$$\sigma(z) = \frac{1}{1+e^{-z}} = \frac{e^z}{e^z+1} = \frac{1+e^z-1}{1+e^z} = 1-\sigma(-z)$$

$$\Rightarrow \boxed{\sigma(-z) = 1-\sigma(z)}$$

- Derivative

$$\boxed{\frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z))}$$

$$\frac{d\sigma(z)}{dz} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}\frac{1}{e^z+1} = \sigma(z)\sigma(-z) = \sigma(z)(1-\sigma(z))$$

Taking the gradient

$$L_{CE} = -\sum_{i=1}^{n} [y^{(i)} \log \sigma(w \cdot x^{(i)} + b) + (1 - y^{(i)}) \log(1 - \sigma(w \cdot x^{(i)} + b))]$$

Consider cross-entropy loss for the ith instance / data point with $z = w \cdot x^{(i)} + b$

$$\frac{dL_{CE}^{(i)}}{dw_j} = \frac{dL_{CE}^{(i)}}{dz}\frac{dz}{dw_j} \qquad \frac{dL_{CE}^{(i)}}{db} = \frac{dL_{CE}^{(i)}}{dz}\frac{dz}{db}$$

$$\frac{dL_{CE}^{(i)}}{dz} = -\left[y^{(i)}\frac{\sigma'(z)}{\sigma(z)} + (1 - y^{(i)})\frac{-\sigma'(z)}{1 - \sigma(z)}\right] \leftarrow \frac{d\sigma(z)}{dz} = \sigma(z)(1 - \sigma(z))$$

$$= -\left[y^{(i)}\frac{\sigma(z)(1 - \sigma(z))}{\sigma(z)} + (1 - y^{(i)})\frac{-\sigma(z)(1 - \sigma(z))}{1 - \sigma(z)}\right]$$

$$= -\left[y^{(i)}(1 - \sigma(z))) + (1 - y^{(i)})(-\sigma(z))\right] = -\left[y^{(i)} - \sigma(z)\right] = \sigma(z) - y^{(i)}$$

$$\boxed{\frac{dL_{CE}^{(i)}}{dz} = \sigma(z) - y^{(i)}}$$

Taking the gradient

$$L_{CE} = -\sum_{i=1}^{n} [y^{(i)} \log \sigma(w \cdot x^{(i)} + b) + (1 - y^{(i)}) \log(1 - \sigma(w \cdot x^{(i)} + b))]$$

$$\frac{dL_{CE}^{(i)}}{dz} = \sigma(z) - y^{(i)}$$

$$z = w \cdot x^{(i)} + b$$

$$\frac{dL_{CE}^{(i)}}{dw_j} = \frac{dL_{CE}^{(i)}}{dz}\frac{dz}{dw_j} = \left(\sigma(w \cdot x^{(i)} + b) - y^{(i)}\right) x_j^{(i)}$$

$$\frac{dz}{dw_j} = x_j^{(i)}$$

$$\frac{dL_{CE}^{(i)}}{db} = \frac{dL_{CE}^{(i)}}{dz}\frac{dz}{db} = \left(\sigma(w \cdot x^{(i)} + b) - y^{(i)}\right)$$

$$\frac{dz}{db} = 1$$

Stochastic Gradient Descent

- Online optimization
- Compute loss and minimize after each training example

function STOCHASTIC GRADIENT DESCENT($L()$, $f()$, x, y) **returns** θ

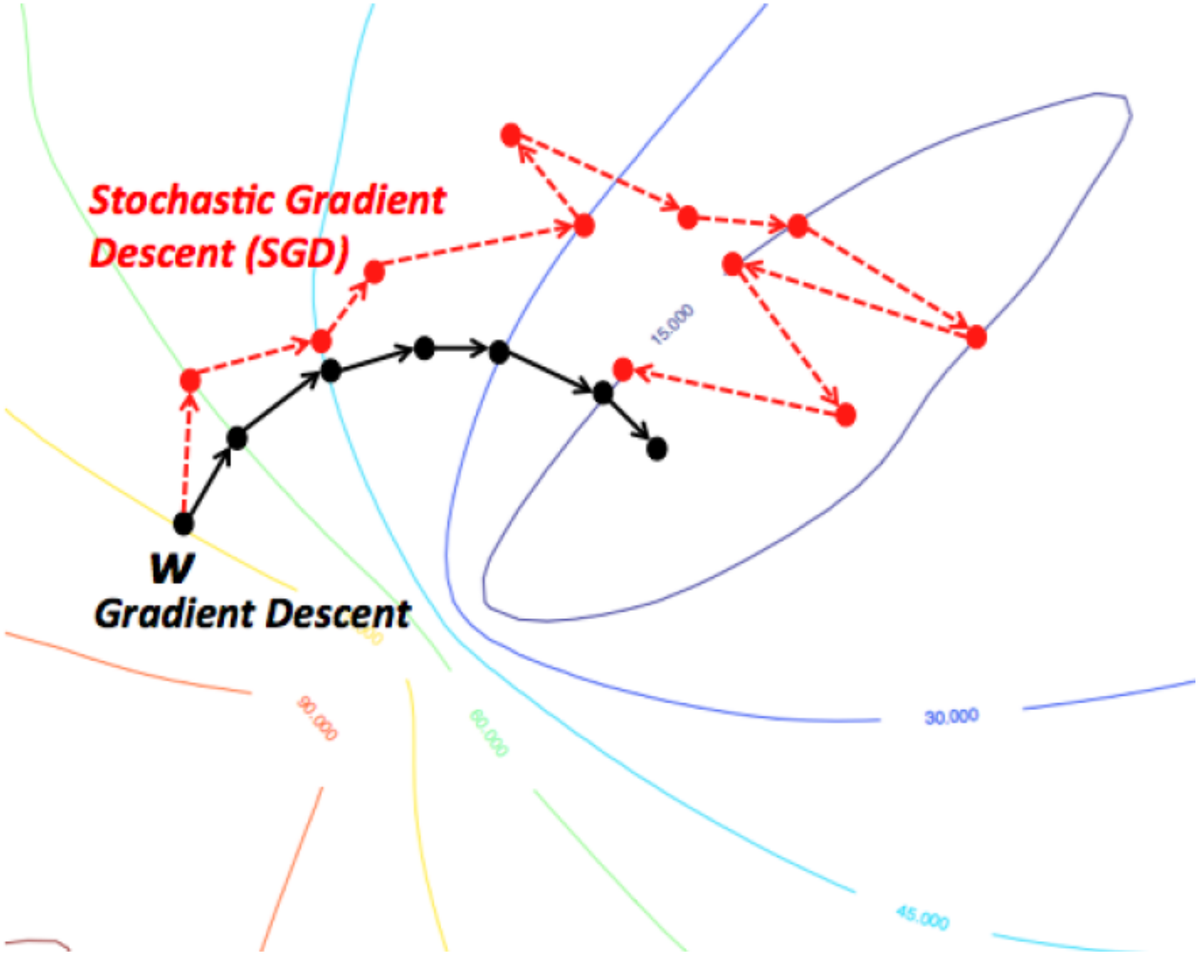
```
    # where: L is the loss function
    #        f is a function parameterized by θ
    #        x is the set of training inputs x^(1), x^(2),..., x^(n)
    #        y is the set of training outputs (labels) y^(1), y^(2),..., y^(n)

θ ← 0
repeat til done    # see caption
   For each training tuple (x^(i), y^(i)) (in random order)
      1. Optional (for reporting):        # How are we doing on this tuple?
         Compute ŷ^(i) = f(x^(i); θ)      # What is our estimated output ŷ?
         Compute the loss L(ŷ^(i), y^(i)) # How far off is ŷ^(i)) from the true output y^(i)?
      2. g ← ∇_θ L(f(x^(i); θ), y^(i))    # How should we move θ to maximize loss?
      3. θ ← θ − η g                      # Go the other way instead
return θ
```

Per Instance Loss

Stochastic Gradient Descent

- Online optimization
- Compute loss and minimize after each training example



Regularization

- Training objective: $\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} \mid x^{(i)})$

- This might fit the training set too well! (including noisy features)

- Poor generalization to the unseen test set — **Overfitting**

- **Regularization** helps prevent overfitting

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} \mid x^{(i)}) - \alpha R(\theta)$$

Penalize large weights

L2 regularization

- $R(\theta) = ||\theta||_2 = \sum_{j=1}^{d} \theta_j^2$

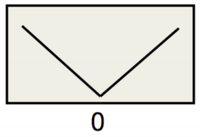
- Euclidean distance of weight vector θ from origin

- L2 regularized objective:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} | x^{(i)}) - \alpha \sum_{j=1}^{d} \theta_j^2$$

L1 Regularization

- $R(\theta) = ||\theta||_1 = \sum_{j=1}^{d} |\theta_j|$



- Manhattan distance of weight vector θ from origin

- L1 regularized objective:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log P(y^{(i)} | x^{(i)}) - \alpha \sum_{j=1}^{d} |\theta_j|$$

L2 vs L1 regularization

- L2 is easier to optimize - simpler derivation
  - L1 is complex since the derivative of $|\theta|$ is not continuous at 0
- L2 leads to many small weights (due to θ^2 term)
  - L1 prefers *sparse* weight vectors with many weights set to 0 (i.e. far fewer features used)