SFU NatLangLab

CMPT 413/713: Natural Language Processing

# Word Embeddings

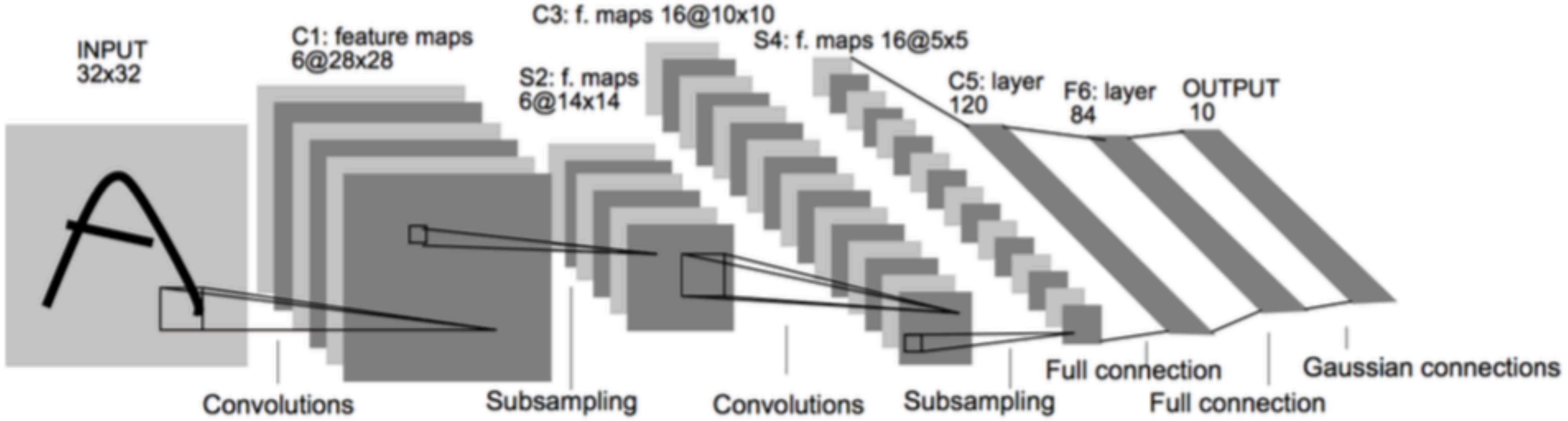
Spring 2024
2024-01–22

Adapted from slides from Dan Jurafsky, Chris Manning, Danqi Chen and Karthik Narasimhan

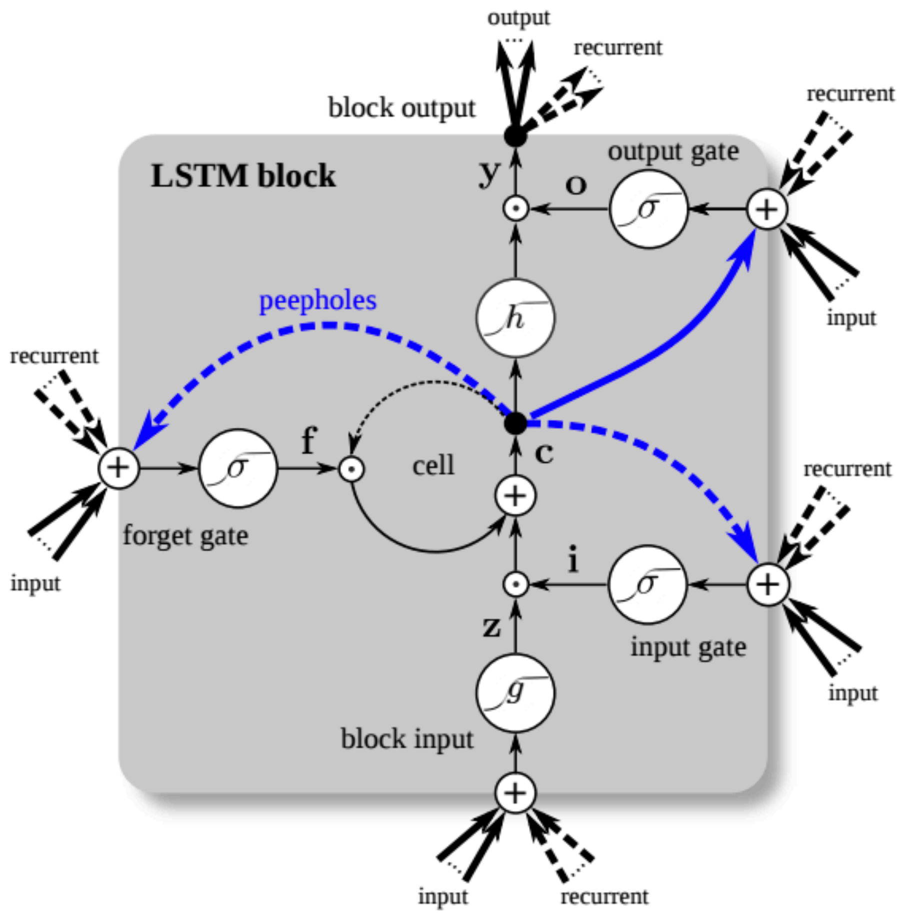
# Neural Networks: Brief history

# NN “dark ages”

- Rosenblatt’s Perceptron (1958)
  - Minsky and Papert (1969) - perceptrons are severely limited
- Neural network algorithms (including backpropagation) date from the 80s
- ConvNets: applied to MNIST by LeCun in 1998




- Long Short-term Memory Networks (LSTMs): Hochreiter and Schmidhuber 1997




- Henderson 2003: neural shift-reduce parser, not SOTA



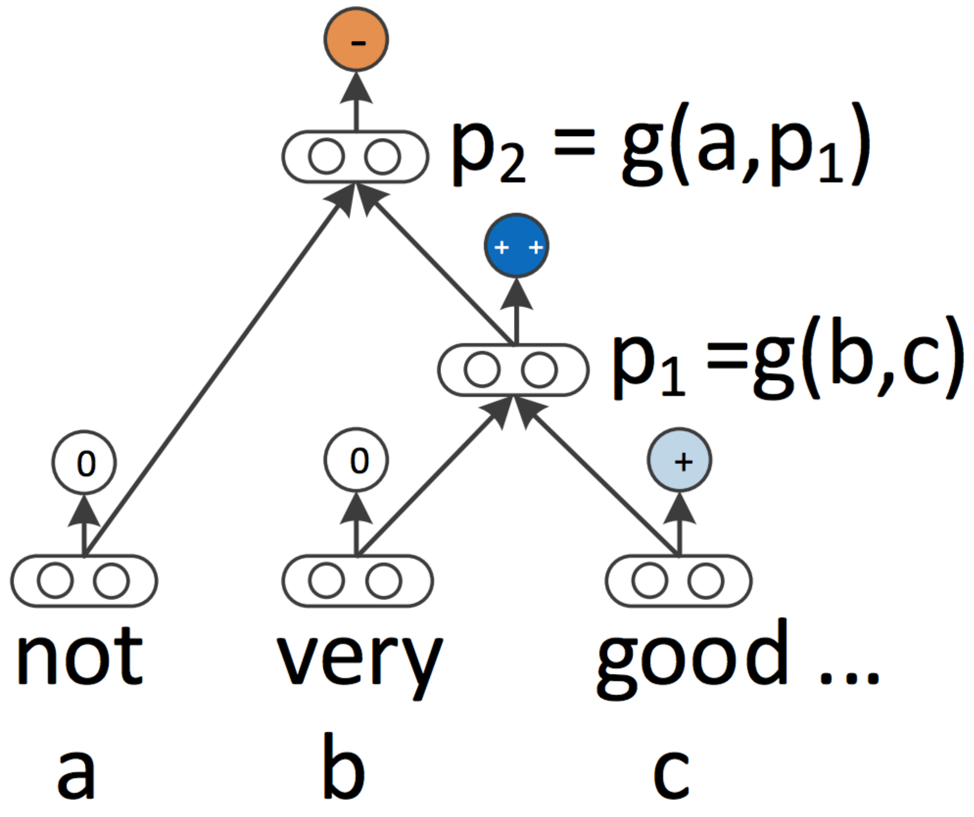
Credits: Greg Durrett

# 2008-2013: A glimmer of light

- Collobert and Weston 2011: "**NLP (almost) from Scratch**"
  - Feedforward NNs can replace "feature engineering"
  - 2008 version was marred by bad experiments, claimed SOTA but wasn't, 2011 version tied SOTA

- Krizhevskey et al, 2012: AlexNet for ImageNet Classification

- Socher 2011-2014: tree-structured RNNs working okay




Credits: Greg Durrett

# 2014: Stuff starts working

- Kim (2014) + Kalchbrenner et al, 2014: sentence classification
  - ConvNets work for NLP!
- Sutskever et al, 2014: sequence-to-sequence for neural MT
  - LSTMs work for NLP!
- Chen and Manning 2014: dependency parsing
  - Even feedforward networks work well for NLP!
- 2015: explosion of neural networks for everything under the sun



Credits: Greg Durrett

# Why didn't they work before?

- **Datasets too small**: for MT, not really better until you have 1M+ parallel sentences (and really need a lot more)

- **Optimization not well understood**: good initialization, per-feature scaling + momentum (Adagrad/Adam) work best out-of-the-box
  - Regularization: dropout is pretty helpful
  - Computers not big enough: can't run for enough iterations

- Inputs: need **word embeddings** to represent continuous semantics



Credits: Greg Durrett

# Representation learning

- Most NLP works in the past focused on human-designed representations and input features

| Var | Definition | Value in Fig. 5.2 |
|---|---|---|
| $x_1$ | count(positive lexicon) $\in$ doc) | 3 |
| $x_2$ | count(negative lexicon) $\in$ doc) | 2 |
| $x_3$ | $\begin{cases} 1 & \text{if "no"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$ | 1 |
| $x_4$ | count(1st and 2nd pronouns $\in$ doc) | 3 |
| $x_5$ | $\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$ | 0 |
| $x_6$ | log(word count of doc) | $\ln(64) = 4.15$ |

- **Representation learning** attempts to automatically learn good features and representations

- **Deep learning** attempts to learn multiple levels of representation on increasing complexity/abstraction

One example: word embeddings

# How to represent words?

In traditional NLP, we regard words as discrete symbols:
hotel, conference, motel — a localist representation

one 1, the rest 0's

Words can be represented by one-hot vectors:

Each word is one dimension!

hotel = [0 0 0 0 0 0 0 0 0 0 0 **1** 0 0 0 0]
motel = [0 0 0 **1** 0 0 0 0 0 0 0 0 0 0 0 0]

Vector dimension = number of words in vocabulary (e.g., 500,000)

There is no way to encode similarity of words in these vectors!

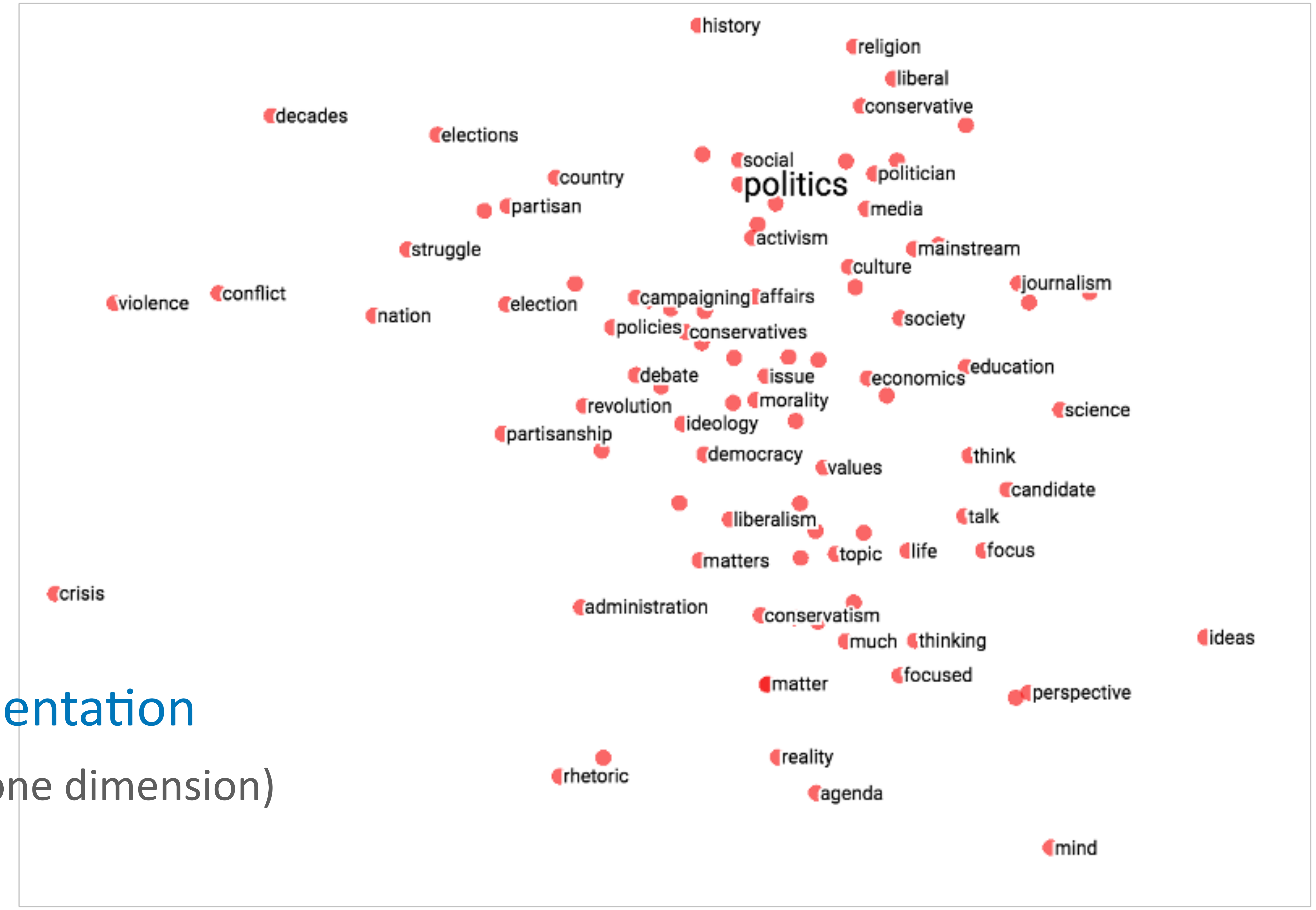
# Word vectors

Continuous space for words:
Similar words closer to each other

$$\text{employees} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 10.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$

Distributed representation

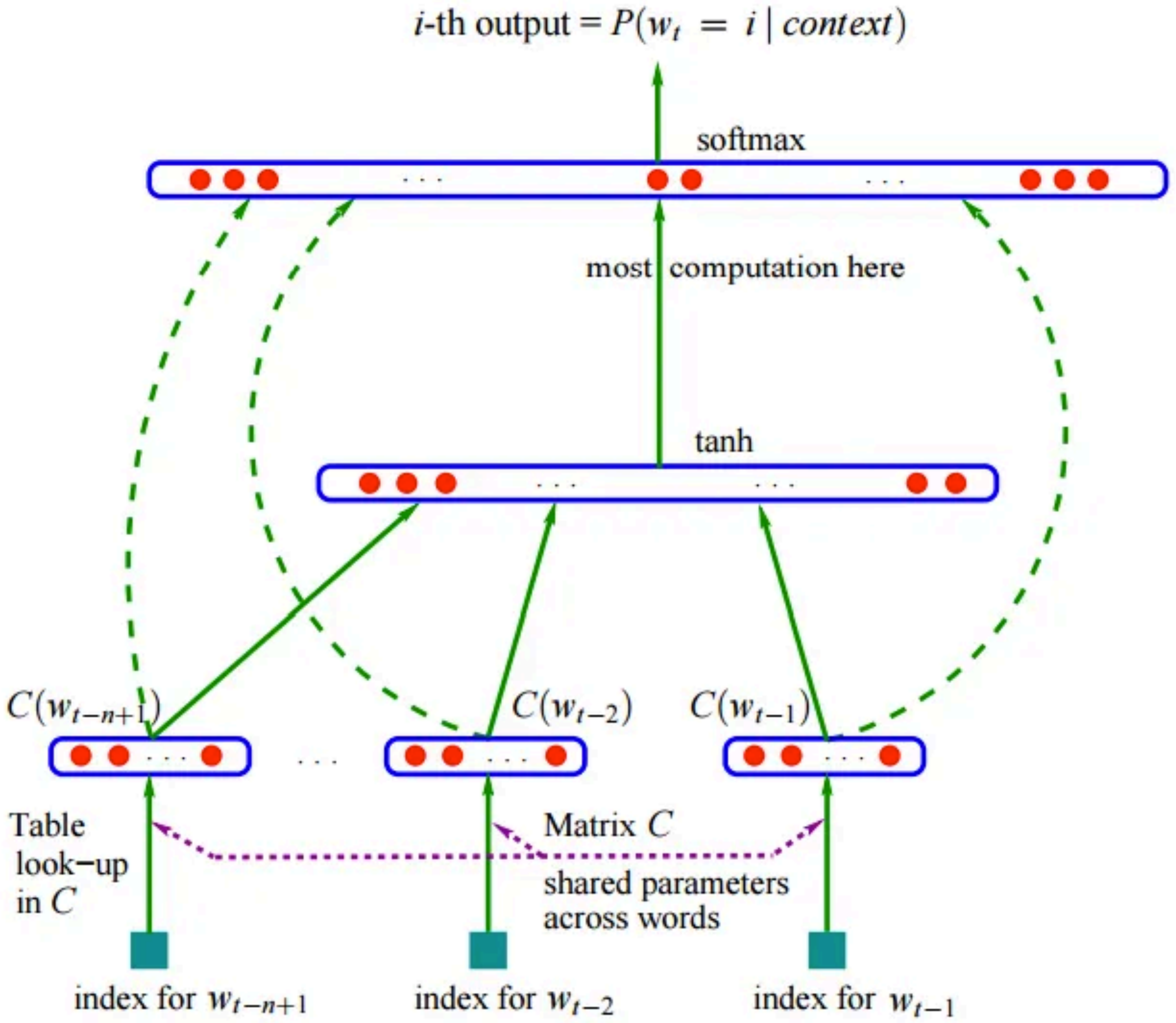
(Meaning not isolated to one dimension)



# Neural Networks with Word Embeddings

# Feedforward Neural LMs

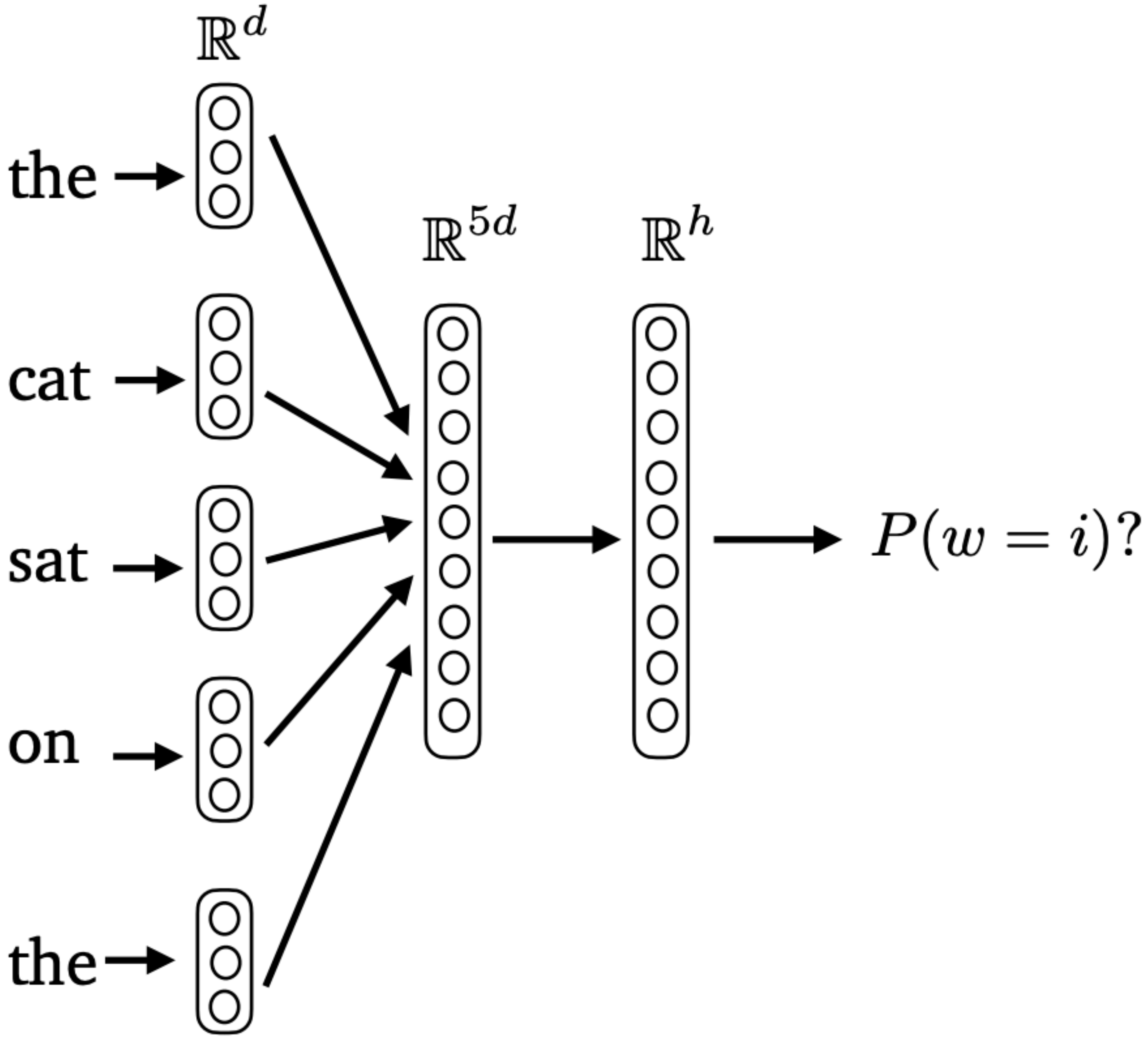
- N-gram models: $P(\text{mat}|\text{the cat sat on the})$




(Bengio et 2003): A Neural Probabilistic Language Model

# Feedforward Neural LMs

- P(mat | the cat sat on the) = ?




- Input layer (context size n = 5):

$$\mathbf{x} = [\mathbf{e}_{\text{the}}; \mathbf{e}_{\text{cat}}; \mathbf{e}_{\text{sat}}; \mathbf{e}_{\text{on}}; \mathbf{e}_{\text{the}}] \in \mathbb{R}^{dn}$$

concatenate word embeddings

- Hidden layer

$$\mathbf{h} = \tanh(\mathbf{W}\mathbf{x} + \mathbf{b}) \in \mathbb{R}^h$$

- Output layer (softmax)

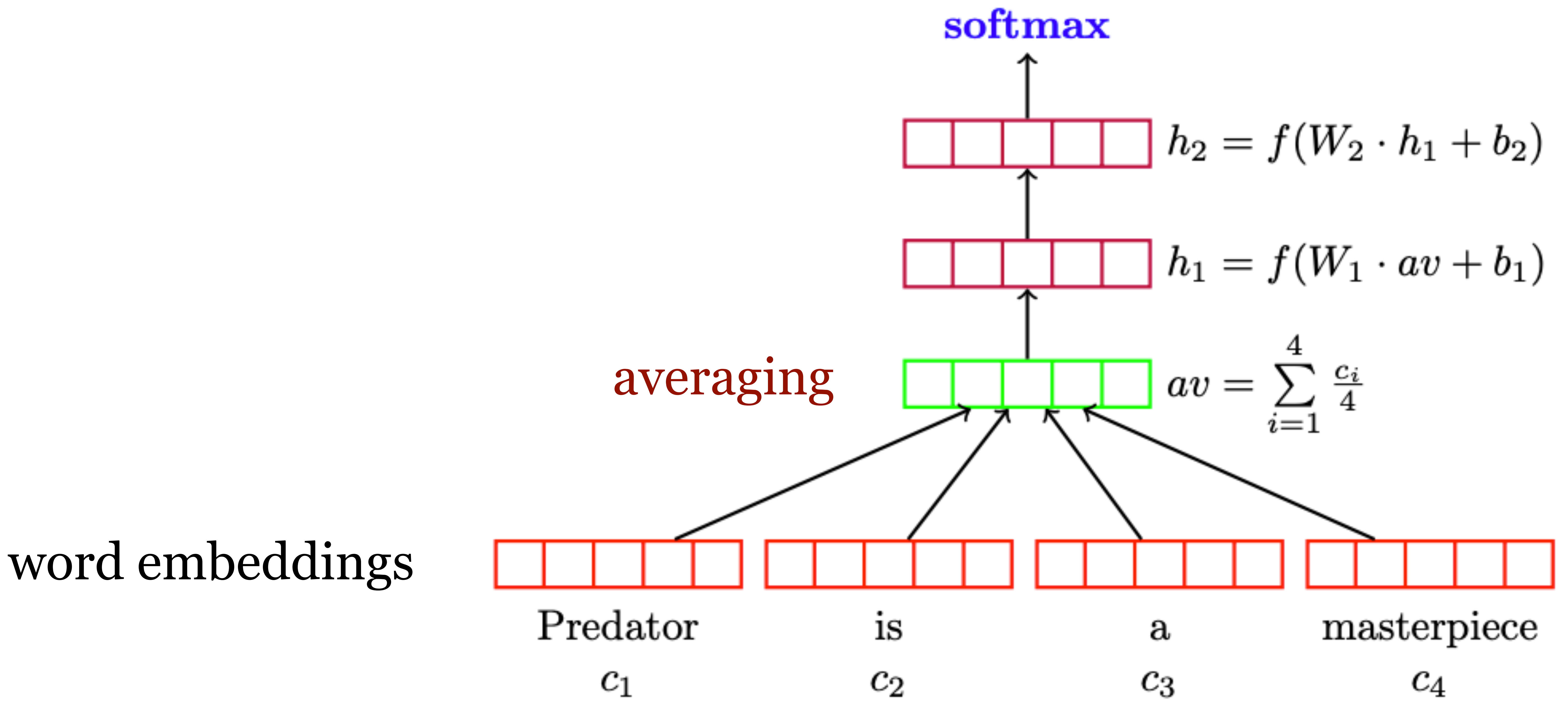
$$\mathbf{z} = \mathbf{U}\mathbf{h} \in \mathbb{R}^{|V|}$$

$$P(w = i \mid \text{context}) = softmax_i(\mathbf{z})$$

(Bengio et 2003): A Neural Probabilistic Language Model

# Neural Bag-of-Words (NBOW)

- Deep Averaging Networks (DAN) for Text Classification

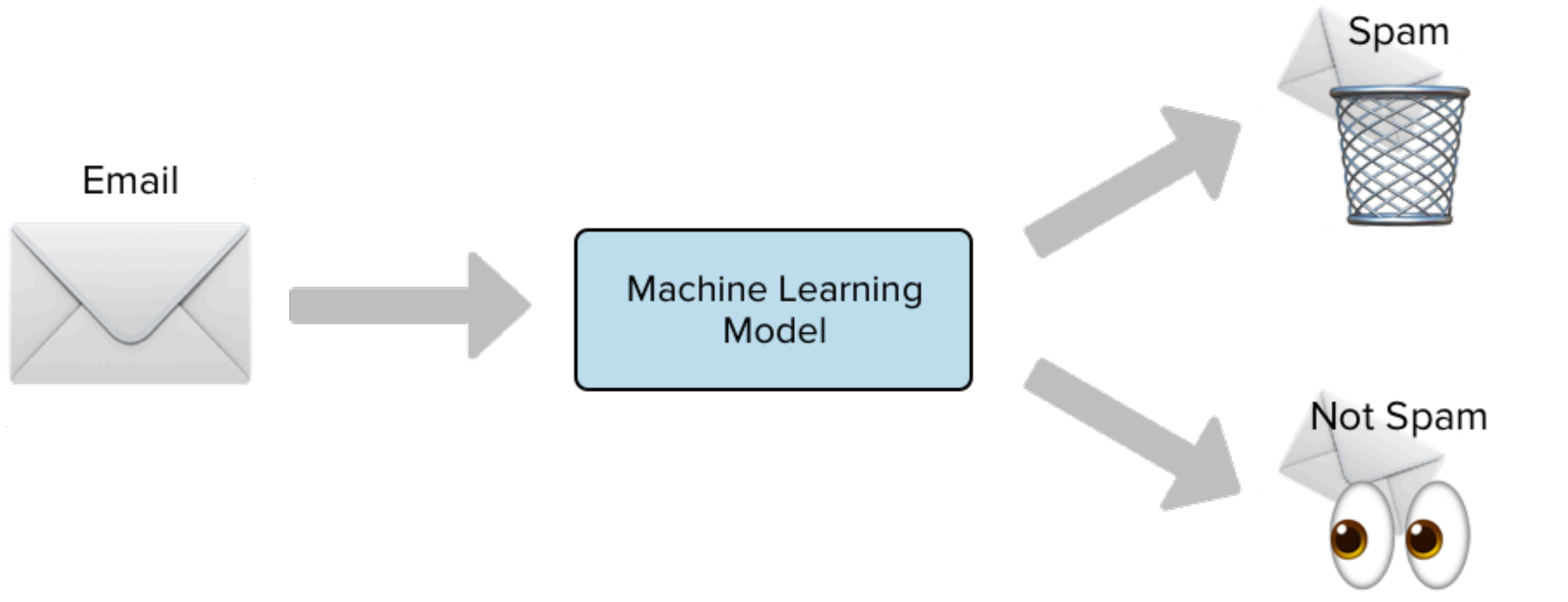


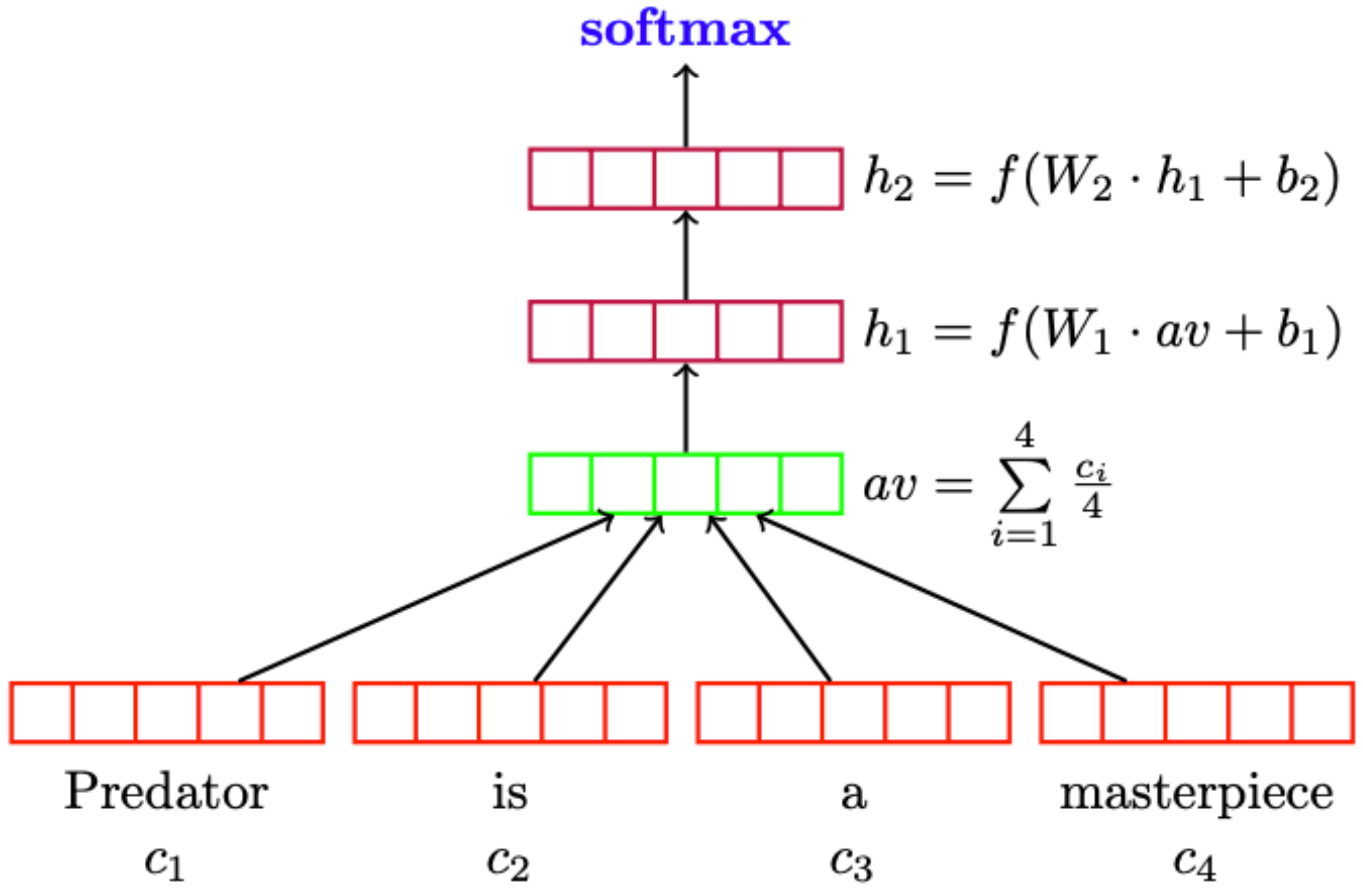

(Iyyer et 2015): Deep Unordered Composition Rivals Syntactic Methods for Text Classification

# Where do these word embeddings come from?

# Task specific training

- Assume you have annotated data specific to a task
- Initialize with random vectors
- Lookup table from word to vector
- Train your classifier
    - Classifier parameters are updated during training
    - These parameters include the word vectors!
    - After training, you get word vectors that are good for your task!



# Pretraining and task-specific fine-tuning

## Pretraining

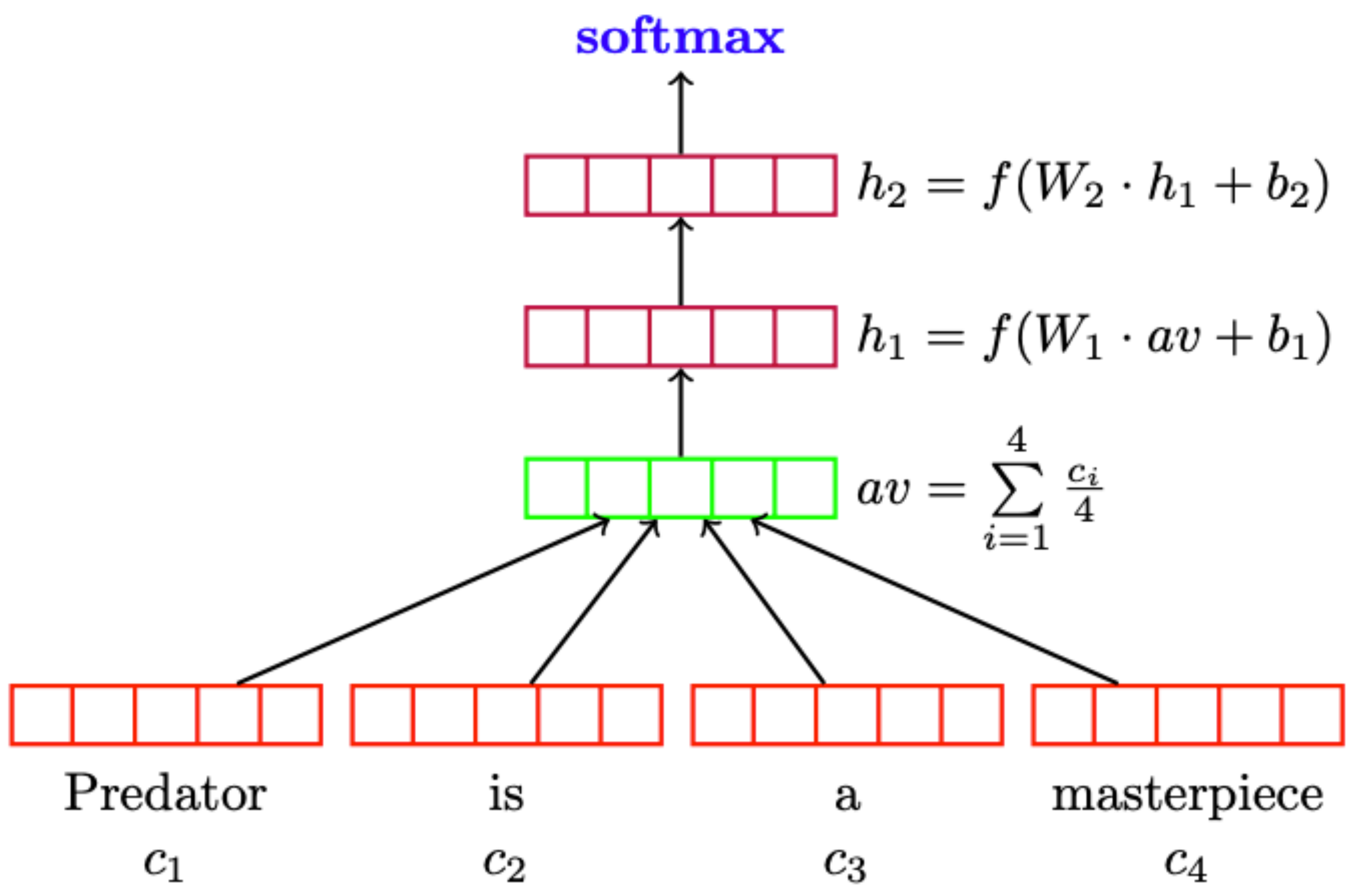
- Big pile of unlabeled text data!
- Lots of resources to train!

## Task-specific fine-tuning

- Annotated data specific to a task (usually small)
- Initialize with pre-trained model

# Summary of three options

- **Random + train** - Initialize with random embeddings and learn them when you train your classifier.
- **Pretrain + fixed** - Initialize with pretrained embeddings + keep them fixed
- **Pretrain + fine-tune** - Initialize with pretrained embeddings and then allow embedding weights to change as classifier is trained



# How does this pretraining work?

Big pile of unlabeled text data!

# Representing words by their context

**Distributional hypothesis**: words that occur in similar contexts tend to have similar meanings

J.R.Firth 1957

- "You shall know a word by the company it keeps"
- One of the most successful ideas of modern statistical NLP!

*…government debt problems turning into* ***banking*** *crises as happened in 2009…*

*…saying that Europe needs unified* ***banking*** *regulation to replace the hodgepodge…*

*…India has just given its* ***banking*** *system a shot in the arm…*

These context words will represent ***banking***.

# Distributional hypothesis

"tejuino"

C1: A bottle of ___ is on the table.

C2: Everybody likes ___.

C3: Don't have ___ before you drive.

C4: We make ___ out of corn.

# Distributional hypothesis

"words that occur in similar contexts tend to have similar meanings"

C1: A bottle of ____ is on the table.

C2: Everybody likes ____.

C3: Don't have ____ before you drive.

C4: We make ____ out of corn.

| | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| tejuino | 1 | 1 | 1 | 1 |
| loud | 0 | 0 | 0 | 0 |
| motor-oil | 1 | 0 | 0 | 0 |
| tortillas | 0 | 1 | 0 | 1 |
| choices | 0 | 1 | 0 | 0 |
| wine | 1 | 1 | 1 | 0 |

Use as context: other words that appear in a span around the target word

# How are these embeddings learned?

Get embeddings by counting or by predicting (i.e. training a classifier)!

C1: A bottle of ____ is on the table.

C2: Everybody likes ____.

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| choices | 0 | 1 | 0 | 0 |
| wine | 1 | 1 | 1 | 0 |

Use as context: other words that appear in a span around the target word

"words that occur in similar contexts tend to have similar meanings"

# Representing words as vectors

- What we are aiming for:
  - Each word is a vector
  - Similar words are "nearby in space"
- Our first solution: use context vectors to represent the meaning of words

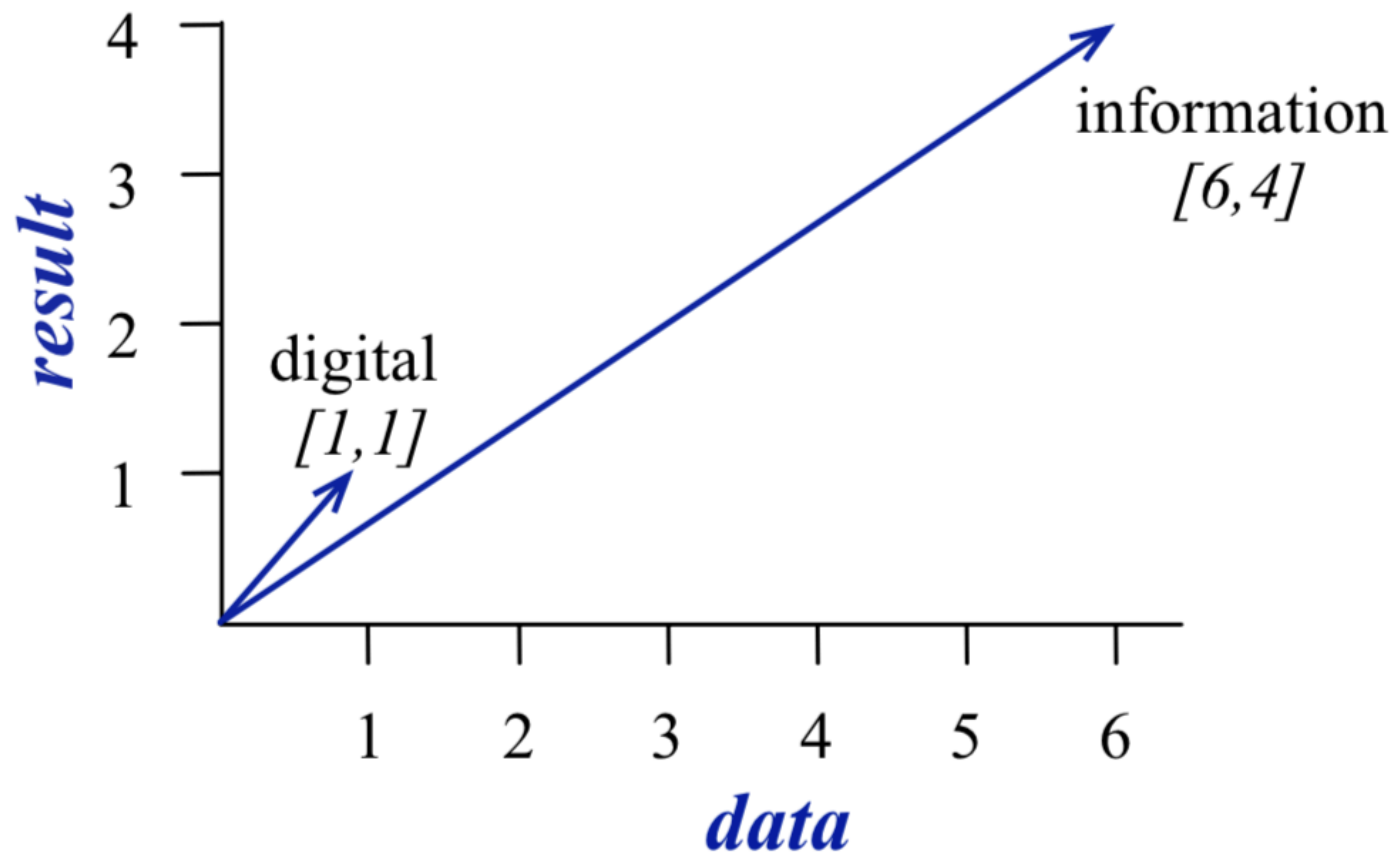
word-word (term-context) co-occurrence matrix

| | | |
|---|---|---|
| sugar, a sliced lemon, a tablespoonful of | **apricot** | jam, a pinch each of, |
| their enjoyment. Cautiously she sampled her first | **pineapple** | and another fruit whose taste she likened |
| well suited to programming on the digital | **computer**. | In finding the optimal R-stage policy from |
| for the purpose of gathering data and | **information** | necessary for the study authorized in the |

| | aardvark | computer | data | pinch | result | sugar | ... |
|---|---|---|---|---|---|---|---|
| apricot | 0 | 0 | 0 | 1 | 0 | 1 | |
| pineapple | 0 | 0 | 0 | 1 | 0 | 1 | |
| digital | 0 | 2 | 1 | 0 | 1 | 0 | |
| information | 0 | 1 | 6 | 0 | 4 | 0 | |

Example from: Dan Jurafsky

# Can measure similarity of words




$$cos(\mathbf{u},\mathbf{v}) = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

$$cos(\mathbf{u},\mathbf{v}) = \frac{\sum_{i=1}^{V} u_i v_i}{\sqrt{\sum_{i=1}^{V} u_i^2}\sqrt{\sum_{i=1}^{V} v_i^2}}$$

# Problem with raw frequencies

**Problem**: using raw frequency counts is not always very good..

- if sugar appears a lot near apricot, that's useful information.
- But overly frequent words like *the*, *it*, or *they* are not very informative about the context

**Solution**: let's weight the counts!

- TF-IDF = Traditional method used in document retrieval
- PPMI = Positive Pointwise Mutual Information

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# Tf-idf

**Term-document matrix**

count$(t, d)$ = count of times term $t$ occurred in document $d$

| | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---|---|---|---|---|
| battle | 1 | 0 | 7 | 13 |
| good | 114 | 80 | 62 | 89 |
| fool | 36 | 58 | 1 | 4 |
| wit | 20 | 15 | 2 | 3 |

- Documents are represented by the column vectors

# Tf-idf

**Term-document matrix**

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|---|---|---|---|---|
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| fool | 36 | 58 | 1 | 4 |
| wit | 20 | 15 | 2 | 3 |

- Documents are represented by the column vectors
- Words are represented by row vectors

Tf-idf: re-weighing scheme for information retrieval

- Down-weighs frequent words such as *this*, *that*, *the*

# Tf-idf combine two factors

**tf: term frequency**. frequency count (usually log-transformed):

$$\text{tf}_{t,d} = \begin{cases} 1 + \log_{10} \text{count}(t,d) & \text{if } \text{count}(t,d) > 0 \\ 0 & \text{otherwise} \end{cases}$$

**idf: inverse document frequency**:

Total # of docs in collection

$$\text{idf}_i = \log\left(\frac{N}{\text{df}_i}\right)$$

There are variations on the exact formulation of tf and idf

# of docs that have word i

Words like "the" or "it" will have very low idf

+1 (avoid 0 in denominator)

tf-idf value for word t in document d

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

# Raw counts vs tf-idf

| Word | df | idf |
|---|---|---|
| Romeo | 1 | 1.57 |
| salad | 2 | 1.27 |
| Falstaff | 4 | 0.967 |
| forest | 12 | 0.489 |
| battle | 21 | 0.246 |
| wit | 34 | 0.037 |
| fool | 36 | 0.012 |
| good | 37 | 0 |
| sweet | 37 | 0 |

37 plays total

## Raw Counts

| | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---|---|---|---|---|
| battle | 1 | 0 | 7 | 13 |
| good | 114 | 80 | 62 | 89 |
| fool | 36 | 58 | 1 | 4 |
| wit | 20 | 15 | 2 | 3 |

## Tf-idf

| | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---|---|---|---|---|
| battle | 0.074 | 0 | 0.22 | 0.28 |
| good | 0 | 0 | 0 | 0 |
| fool | 0.019 | 0.021 | 0.0036 | 0.0083 |
| wit | 0.049 | 0.044 | 0.018 | 0.022 |

Common words
are down weighted

# Tf-idf summary

- Tf-idf: term-frequency inverse-document frequency
- Re-weighing scheme originally designed for information retrieval
  - Down-weighs frequent words such as *this*, *that*, *the*
- Can be used to
  - measure the similarity between words
  - measure the similarity between documents
  - measure the similarity between a query (mini-document) and a document
  - as features for classifiers
- Useful to know about (good baseline method)
- But not typically used for word embeddings

# Problem with raw frequencies

**Problem**: using raw frequency counts is not always very good..

- if sugar appears a lot near apricot, that's useful information.
- But overly frequent words like *the*, *it*, or *they* are not very informative about the context

**Solution**: let’s weight the counts!

- TF-IDF = Traditional method used in document retrieval
- PPMI = Positive Pointwise Mutual Information

# PPMI

Do two events co-occur more than if they were independent?

PPMI = Positive Pointwise Mutual Information

Joint probability

$$\mathrm{PPMI}(w,c) = \max\left(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0\right)$$

Marginals

PMI ranges from $-\infty$ to $+\infty$

Negative if co-occurs less than if independent

Negative values are problematic so cap bottom to 0

# Computing PPMI on a term-context matrix

- Matrix $F$ with $W$ rows (words) and $C$ columns (contexts)
- $f_{ij}$ is # of times $w_i$ occurs in context $c_j$

| | aardvark | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|---|
| apricot | 0 | 0 | 0 | 1 | 0 | 1 |
| pineapple | 0 | 0 | 0 | 1 | 0 | 1 |
| digital | 0 | 2 | 1 | 0 | 1 | 0 |
| information | 0 | 1 | 6 | 0 | 4 | 0 |

$$ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

**Joint probability**

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

**Marginals**

$$p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

# Computing PPMI on a term-context matrix

count($w_i$, context$_j$)

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

Total count N

**Count(w,context)**

| | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|
| apricot | 0 | 0 | 1 | 0 | 1 |
| pineapple | 0 | 0 | 1 | 0 | 1 |
| digital | 2 | 1 | 0 | 1 | 0 |
| information | 1 | 6 | 0 | 4 | 0 |

$p(w = \text{information}, c = \text{data}) = 6/19 = 0.32$

**p(w,context)** **p(w)**

| | computer | data | pinch | result | sugar | p(w) |
|---|---|---|---|---|---|---|
| apricot | 0.00 | 0.00 | 0.05 | 0.00 | 0.05 | 0.11 |
| pineapple | 0.00 | 0.00 | 0.05 | 0.00 | 0.05 | 0.11 |
| digital | 0.11 | 0.05 | 0.00 | 0.05 | 0.00 | 0.21 |
| information | 0.05 | 0.32 | 0.00 | 0.21 | 0.00 | 0.58 |
| **p(context)** | 0.16 | 0.37 | 0.11 | 0.26 | 0.11 | |

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N}$$

$$p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

$p(w = \text{information}) = 11/19 = 0.58$

$p(c = \text{data}) = 7/19 = 0.37$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

$$pmi(w = \text{information}, c = \text{data}) = \log_2 \frac{0.32}{0.37 \times 0.58} = 0.58$$

**PPMI(w,context)**

| | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|
| apricot | - | - | 2.25 | - | 2.25 |
| pineapple | - | - | 2.25 | - | 2.25 |
| digital | 1.66 | 0.00 | - | 0.00 | - |
| information | 0.00 | 0.57 | - | 0.47 | - |

Actually 0.57 using full precision

**p(w,context)** / **p(w)**

| | computer | data | pinch | result | sugar | p(w) |
|---|---|---|---|---|---|---|
| apricot | 0.00 | 0.00 | 0.05 | 0.00 | 0.05 | 0.11 |
| pineapple | 0.00 | 0.00 | 0.05 | 0.00 | 0.05 | 0.11 |
| digital | 0.11 | 0.05 | 0.00 | 0.05 | 0.00 | 0.21 |
| information | 0.05 | 0.32 | 0.00 | 0.21 | 0.00 | 0.58 |
| **p(context)** | 0.16 | 0.37 | 0.11 | 0.26 | 0.11 | |

$$ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

# Issues with PMI

PMI is biased toward infrequent events

- Very rare words have very high PMI values

Two solutions:

- Weighted PMI: Give rare words slightly higher probabilities
- Use add-one smoothing (which has a similar effect)

# Weighting PMI

Give rare context words slightly higher probability

Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0)$$

$P(c)$ is low, so PPMI is high

Higher $P_\alpha(c) \rightarrow$ lower $\text{PPMI}_\alpha$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

This helps because $P_\alpha(c) > P(c)$ for rare $c$

Consider two events, $P(a) = .99$ and $P(b) = .01$

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97$$

$$P_\alpha(b) = \frac{.01^{.75}}{.99^{.75} + .01^{.75}} = .03$$

# Smoothed PPMI (add-2)

- Use Laplace smoothing
- Alternative to using weighted PPMI

**Add-2 Smoothed Count(w,context)**

| | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|
| apricot | 2 | 2 | 3 | 2 | 3 |
| pineapple | 2 | 2 | 3 | 2 | 3 |
| digital | 4 | 3 | 2 | 3 | 2 |
| information | 3 | 8 | 2 | 6 | 2 |

**p(w,context) [add-2]**

| | computer | data | pinch | result | sugar | **p(w)** |
|---|---|---|---|---|---|---|
| apricot | 0.03 | 0.03 | 0.05 | 0.03 | 0.05 | 0.20 |
| pineapple | 0.03 | 0.03 | 0.05 | 0.03 | 0.05 | 0.20 |
| digital | 0.07 | 0.05 | 0.03 | 0.05 | 0.03 | 0.24 |
| information | 0.05 | 0.14 | 0.03 | 0.10 | 0.03 | 0.36 |
| **p(context)** | 0.19 | 0.25 | 0.17 | 0.22 | 0.17 | |

# PPMI versus add-2 smoothed PPMI

**PPMI(w,context)**

| | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|
| apricot | - | - | 2.25 | - | 2.25 |
| pineapple | - | - | 2.25 | - | 2.25 |
| digital | 1.66 | 0.00 | - | 0.00 | - |
| information | 0.00 | 0.57 | - | 0.47 | - |

**PPMI(w,context) [add-2]**

| | computer | data | pinch | result | sugar |
|---|---|---|---|---|---|
| apricot | 0.00 | 0.00 | 0.56 | 0.00 | 0.56 |
| pineapple | 0.00 | 0.00 | 0.56 | 0.00 | 0.56 |
| digital | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 |
| information | 0.00 | 0.58 | 0.00 | 0.37 | 0.00 |

# Building word vectors by counting

- Build word-word (term-context) co-occurrence matrix

| | computer | data | result | pie | sugar |
|---|---|---|---|---|---|
| **cherry** | 2 | 8 | 9 | 442 | 25 |
| **strawberry** | 0 | 0 | 1 | 60 | 19 |
| **digital** | 1670 | 1683 | 85 | 5 | 4 |
| **information** | 3325 | 3982 | 378 | 5 | 13 |

Raw counts

| | computer | data | result | pie | sugar |
|---|---|---|---|---|---|
| **cherry** | 0 | 0 | 0 | 4.38 | 3.30 |
| **strawberry** | 0 | 0 | 0 | 4.10 | 5.51 |
| **digital** | 0.18 | 0.01 | 0 | 0 | 0 |
| **information** | 0.02 | 0.09 | 0.28 | 0 | 0 |

Positive pointwise mutual information

Practically, use smoothed/weighted PPMI to help with rare words having very high PMI values

$$\text{PPMI}(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0)$$

Vectors are very long, sparse. How to get short dense vectors?

# Sparse vs dense vectors

Vectors we get from word-word (term-context) co-occurrence matrix are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

True for both one-hot and PPMI vectors

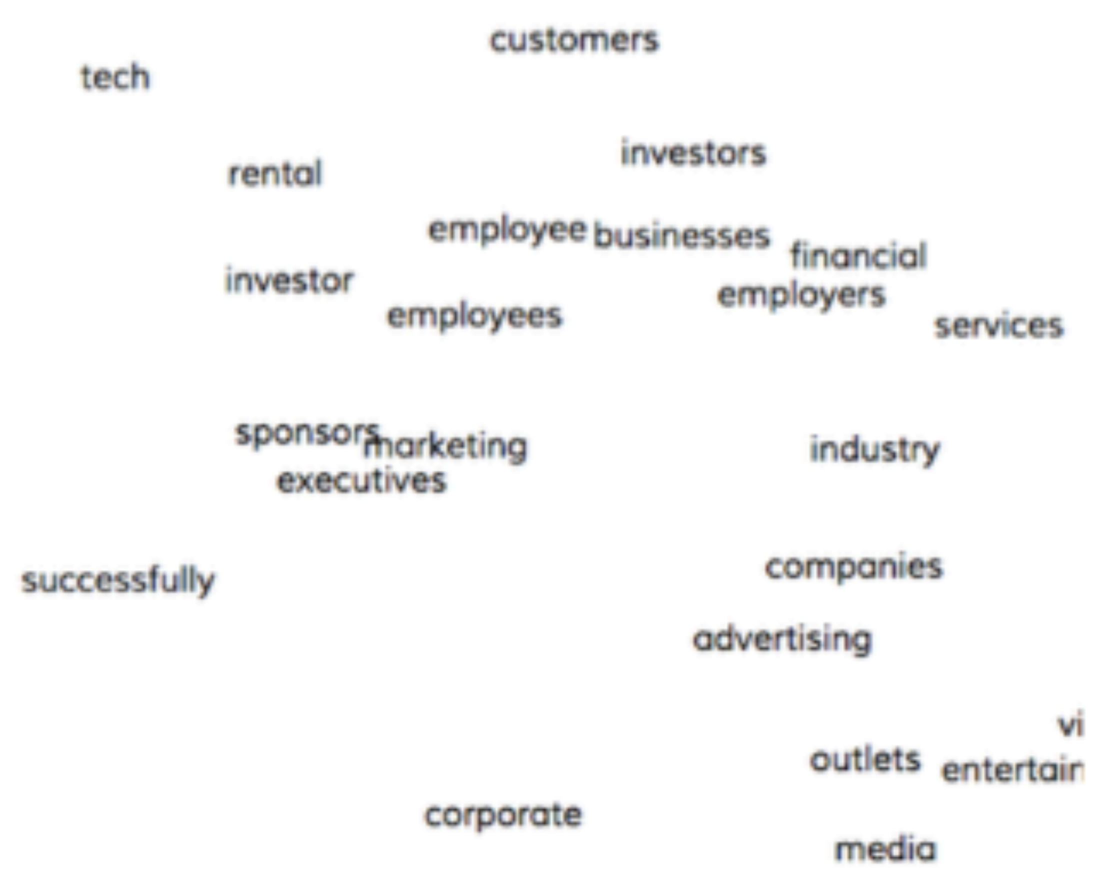
Alternative: we want to represent words as

- **short** (50-300 dimensional)
- **dense** (real-valued) vectors

More memory efficient and easier to work with
Capture similarity between words better

Distributed representation

$$\text{employees} = \begin{pmatrix} 0.286 \\ 0.792 \\ -0.177 \\ -0.107 \\ 10.109 \\ -0.542 \\ 0.349 \\ 0.271 \\ 0.487 \end{pmatrix}$$

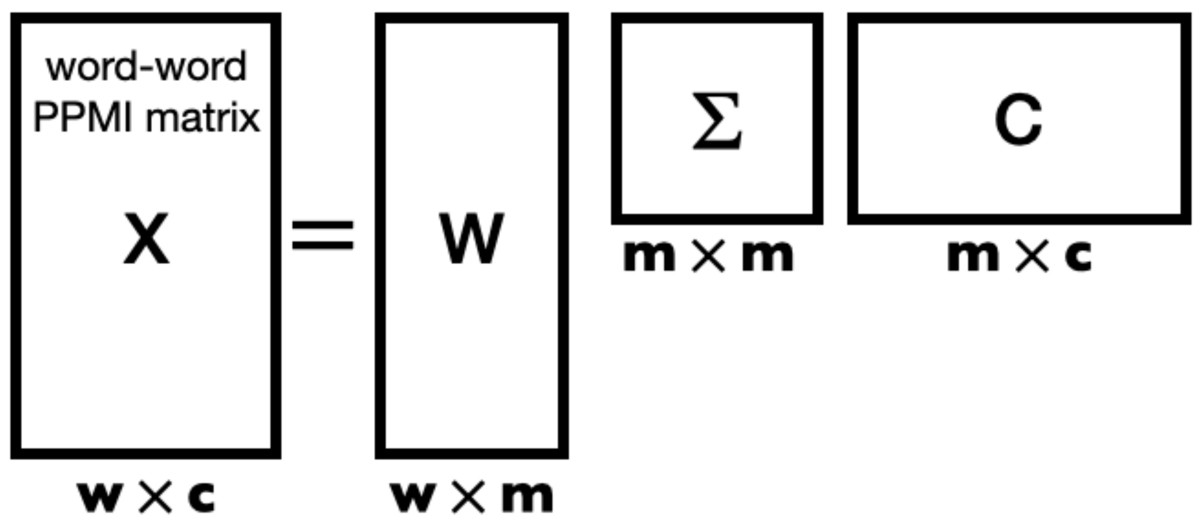


# Why dense vectors?

- Short vectors are easier to use as features in ML systems
- Dense vectors may generalize better than storing explicit counts
- They do better at capturing synonymy
    - $w_1$ co-occurs with "car", $w_2$ co-occurs with "automobile"

- Different methods for getting dense vectors:
    - Singular value decomposition (SVD)
    - word2vec and friends: "learn" the vectors!

SVD




Count based method
(known since the 1990s)

# Using SVD for obtaining dense vectors

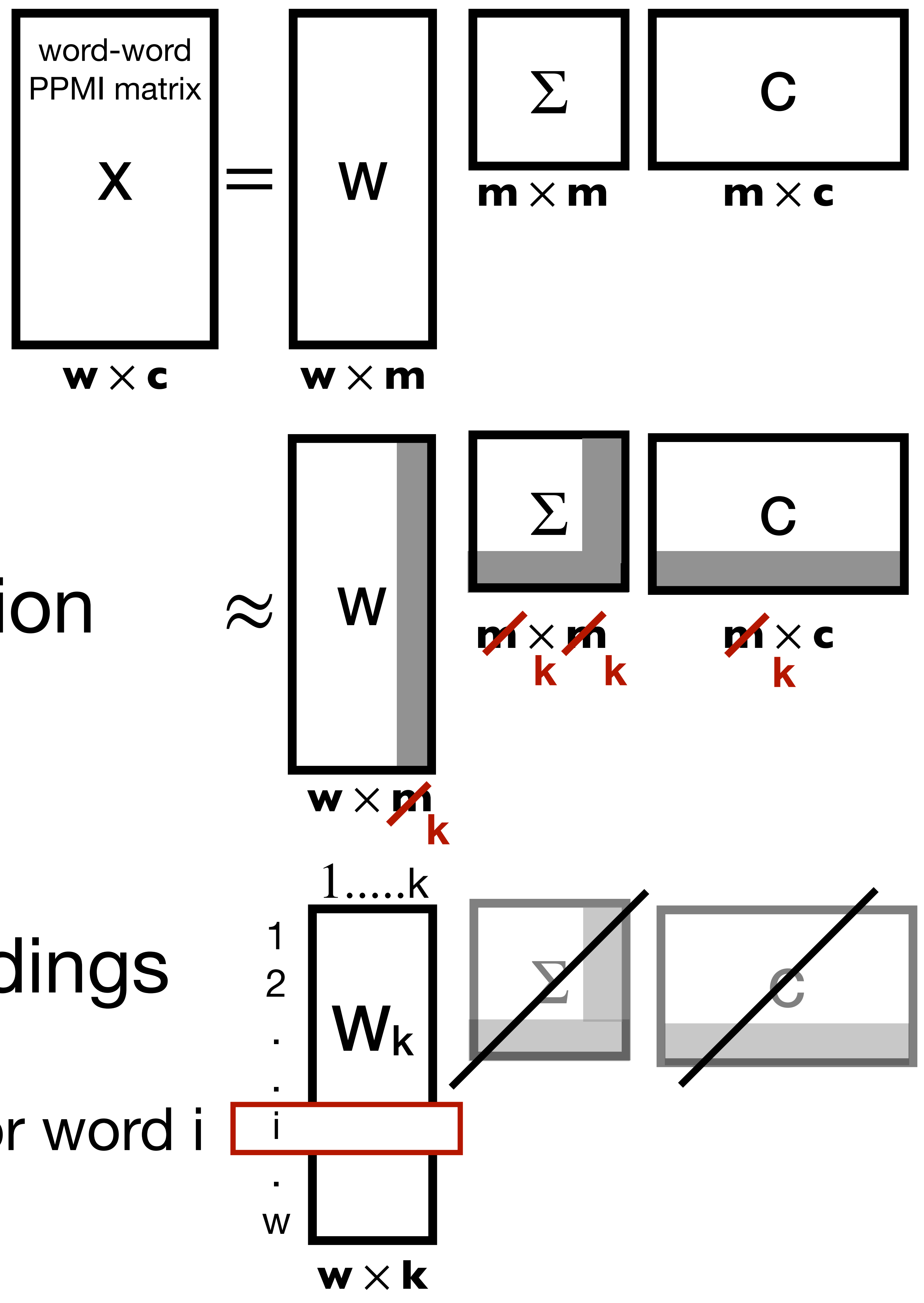
$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top$$

1) SVD

2) Truncation

3) Embeddings

embedding for word i




SVD = Singular value decomposition

$\mathbf{\Sigma}$ is a diagonal matrix with singular values:

$\sigma_1, \ldots, \sigma_i, \ldots, \sigma_m$ where $m$ is the rank of the matrix $\mathbf{X}$

$\mathbf{U} = \mathbf{W}$ has orthonormal columns, $\mathbf{V}^\top = \mathbf{C}$ has orthonormal rows

In our case, $\mathbf{X}$ is a $|V| \times |V|$ matrix. Assume $m = |V|$

Truncation:

- Select the first $k$ columns of $\mathbf{W}$ to get $k$-dimensional row vectors
- Note that the singular values are ordered from largest to smallest, which each singular value representing the variance captured by that dimension
- So the first $k$ dimensions are the dimensions with the most variance

Finally, we take $i$th row of $\mathbf{W}_k$ as the embedding of word $i$

Note there are variants where the word embedding matrix is taken to be

$\mathbf{W}_k\mathbf{\Sigma}^\lambda$ (with common values being $\boxed{\lambda = 1}$, 0.5, or 0)

traditional

# What is wrong with using SVD?

- Computational complexity is high: $O(|V|^3)$
- Cannot be trained as part of a larger model
  - Not a component that can be part of a larger neural network
- Cannot be trained discriminatively for a particular task

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# How are these embeddings learned?

Learn predictor to fill in the blank!

C1: A bottle of ____ is on the table.

- Represent each word as a vector
- Train classifier to predict word using context words.
- During training, the word vector is updated so that it is possible to predict the center word using the context words

| | bottle | likes | before | make | corn |
|---|---|---|---|---|---|
| tejuino | 1 | 1 | 1 | 1 | 1 |
| loud | 0 | 0 | 0 | 0 | 0 |
| motor-oil | 1 | 0 | 0 | 0 | 0 |
| tortillas | 0 | 1 | 0 | 1 | 1 |
| choices | 0 | 1 | 0 | 0 | 0 |
| wine | 1 | 1 | 1 | 0 | 0 |

Use as context: other words that appear in a span around the target word

"words that occur in similar contexts tend to have similar meanings"

# Summary

- Representing words as vectors
  - One-hot vectors vs vectors built using context
  - Cosine similarity for measuring similarity between words
- Using context to represent words
  - Distributional hypothesis:

    words that occur in similar contexts tend to have similar meanings
- Co-occurrence matrix
  - Raw counts
  - TF-IDF (term-frequency inverse-document-frequency)
  - PPMI (positive pointwise mutual information)
- Dense vectors via SVD or learned by predicting the missing word